Section 8 – 2: Multiplying or Dividing Rational Expressions

Multiplying Fractions

The basic rule for multiplying fractions is to **multiply the numerators** together and **multiply the denominators** together

$$\frac{a}{b} \bullet \frac{c}{d} = \frac{a \bullet c}{b \bullet d}$$

and then reduce the answer.

It is often much faster if we **reduce the factors before we multiply**. This is done by **factoring the numerators and denominators** and then **canceling all common factors.** We then multiply the remaining factors to get the final answer.

Example 1

$$\frac{6}{5} \bullet \frac{15}{4}$$

$$= \frac{2 \bullet 3}{5} \bullet \frac{3 \bullet 5}{2 \bullet 2}$$ factor each numerator
factor each denominator

$$= \frac{2 \bullet 3 \bullet 3 \bullet 5}{5 \bullet 2 \bullet 2}$$ Rewrite the factors as a single fraction

$$= \frac{\cancel{2} \bullet 3 \bullet 3 \bullet \cancel{5}}{\cancel{5} \bullet \cancel{2} \bullet 2}$$ Cancel all common factors

$$= \frac{9}{2}$$ Write the remaining factors in fraction form

Multiplying Rational Expressions

We multiply Rational Expressions the same way that we multiply fractions We **completely factor** the polynomials in the numerators and denominators and then cancel out all the common factors.

1. **Completely factor the polynomials** in the numerators and denominators.
2. Rewrite the fraction with the **monomial terms first** and the **binomial terms last.**
3. Cancel out the common binomial factors and use the quotient rule to reduce the monomial terms.
4. Write the remaining factors in fraction form.

Example 2

$$\frac{2x+8}{x-3} \bullet \frac{5x-15}{8x+16}$$

$$= \frac{2(x+4)}{(x-3)} \bullet \frac{5(x-3)}{8(x+2)}$$ factor each numerator; factor each denominator

$$= \frac{10\ (x+4)\ (x-3)}{8\ (x-3)\ (x+2)}$$ Rewrite the fraction with the monomional terms first and the binominal terms last.

cancel the common binominal factors $\frac{(x-3)}{(x-3)}$

and reduce $\frac{10}{8}$ to get $\frac{5}{4}$

$$= \frac{\cancel{10}^{5}\ (x+4)\ \cancel{(x-3)}}{\cancel{8}^{4}\ \cancel{(x-3)}(x+2)} = \frac{5\ (x+4)}{4\ (x+2)}$$

Note: You **CANNOT** cancel a **part of a binomial term**. The 4 in the (x + 4) term in the numerator **CANNOT** be canceled with the 4 in the denominator.

You **CANNOT** cancel a **part of a binomial term**. The x in the (x + 4) term in the numerator **CANNOT** be canceled with the x in the(x + 2) term in the denominator.

The only factor that could cancel the binomial term (x + 4) in the numerator is the exact binomial term (x + 4) in the denominator.

Each part of a monomial term i**s a separate factor**. You **CAN cancel parts of the monomial term in the numerator with parts of the monomial term in the denominator using the quotient rule.**

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Example 3

$$\frac{3x-3}{x^2} \bullet \frac{5x^2-10x}{6x-6}$$

$$= \frac{3(x-1)}{x^2} \bullet \frac{5x(x-2)}{6(x-1)}$$ factor each numerator, factor each denominator

$$= \frac{15\ \ x\ \ (x-1)\ (x-2)}{6\ \ x^2\ \ (x-1)}$$ Rewrite the fraction with the monomional terms first and the binominal terms last.

cancel the common binominal factors $\frac{(x-1)}{(x-1)}$

and reduce $\frac{15x}{6x^2}$ to $\frac{5}{2x}$ using the quoient rule

$$= \frac{\not{15}^{5}\ \ \not{x}\ \ \cancel{(x-1)}\ (x-2)}{\not{6}^{2}\ \ \not{x^2}_{x}\ \ \cancel{(x-1)}} = \frac{5(x-2)}{2x}$$

Note: You **CANNOT** cancel a part of a binomial term. The 2 in the (x – 2) term in the numerator **CANNOT** be canceled with the 2 in the denominator.

You **CANNOT** cancel a part of a binomial term. The x in the (x – 2) term in the numerator **CANNOT** be canceled with the x in the 2x term in the denominator.

The only factor that could cancel the binomial term (x – 2) in the numerator is the exact binomial term (x – 2) in the denominator.

Each part of a monomial term i**s a separate factor**. You **CAN cancel parts of the monomial term in the numerator with parts of the monomial term in the denominator using the quotient rule.**

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Example 4

$$= \frac{4x^2y^2}{5x-15} \bullet \frac{x^2-9}{12x^4y}$$

$$= \frac{4x^2y^2}{5(x-3)} \bullet \frac{(x+3)(x-3)}{12x^4y}$$ factor each numerator
factor each denominator

$$= \frac{4x^2y^2\ (x+3)\ (x-3)}{60x^4y\ (x-3)}$$ Rewrite the fraction with the monomional terms first and the binominal terms last.

cancel the common binominal factors $\frac{(x-3)}{(x-3)}$

and reduce $\frac{4x^2y^2}{60x^4y}$ to $\frac{y}{15x^2}$ using the quoient rule

$$\frac{\cancel{4}^{1}\ \cancel{x^2}\ \cancel{y^2}^{\,y}\ (x+3)\ \cancel{(x-3)}}{\cancel{60}^{15}\ \cancel{x^4}_{x^2}\ \cancel{y}\ \cancel{(x-3)}} = \frac{y(x+3)}{15x^2}$$

Example 5

$$\frac{4x-4}{x^3-4x^2} \bullet \frac{2x^2-8x}{x^2-1}$$

$$= \frac{4(x-1)}{x^2(x-4)} \bullet \frac{2x(x-4)}{(x+1)(x-1)}$$ factor each numerator
factor each denominator

$$= \frac{8\ x\ (x-1)\ (x-4)}{x^2\ (x-4)\ (x+1)\ (x-1)}$$ Rewrite the fraction with the monomional terms first.
and the binominal terms last.

cancel the common binominal factors $\frac{(x-1)}{(x-1)}$ and $\frac{(x-4)}{(x-4)}$

and reduce $\frac{8x}{x^2}$ to $\frac{8}{x}$ using the quoient rule

$$\frac{8\ \cancel{x}\ \cancel{(x-1)}\ \cancel{(x-4)}}{\cancel{x^2}_{x}\ \cancel{(x-4)}\ (x+1)\ \cancel{(x-1)}} = \frac{8}{x(x+1)}$$

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Dividing Rational Expressions

We divide Rational Expressions like

$$\frac{a}{b} \div \frac{c}{d}$$

by changing the division operation into a multiplication operation.

We do this by **inverting** (flipping over) **the fraction to the right of the division sign** and **changing the operation to multiplication.**

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c}$$

1. Invert the second fraction and **change the operation to multiplication.**
2. **Completely factor the polynomials** in the numerators and denominators.
3. Rewrite the fraction with the **monomial terms first** and the **binomial terms last.**
4. Cancel out the common binomial factors and use the quotient rule to reduce the monomial terms.
5. Write the remaining factors in fraction form.

Example 6

$$\frac{4xy}{x^2 - x - 6} \div \frac{2x^3 + 6x^2}{x^2 - 9}$$

Invert the second fraction and multiply

$$= \frac{4xy}{x^2 - x - 6} \bullet \frac{x^2 - 9}{2x^3 + 6x^2}$$

$$= \frac{4xy}{(x-3)(x+2)} \bullet \frac{(x+3)(x-3)}{2x^2(x+3)}$$ factor each numerator factor each denominator

$$= \frac{4xy\ (x+3)\ (x-3)}{2x^2(x-3)(x+2)\ (x+3)}$$ Rewrite the fraction with the monomional terms first and the binominal terms last.

cancel the common binominal factors $\frac{(x-3)}{(x-3)}$ and $\frac{(x+3)}{(x+3)}$

and reduce $\frac{4xy}{2x^2}$ to $\frac{2y}{x}$ using the quoient rule

$$= \frac{\cancel{4}^2\ \cancel{x}\ y\ \cancel{(x+3)}\ \cancel{(x-3)}}{\cancel{2}^1\ \cancel{x^2}_{x}\ \cancel{(x-3)}(x+2)\ \cancel{(x+3)}} = \frac{2y}{x(x+2)}$$

Example 7

$$\frac{x^2-x-6}{x^2+3x+2} \div \frac{5x-15}{x^2+2x}$$

Invert the second fraction and multiply

$$= \frac{x^2-x-6}{x^2+3x+2} \bullet \frac{x^2+2x}{5x-15}$$

$$= \frac{(x-3)(x+2)}{(x+2)(x+1)} \bullet \frac{x(x+2)}{5(x-3)}$$ factor each numerator, factor each denominator

$$= \frac{x\ (x-3)\ (x+2)(x+2)}{5(x+2)(x+1)\ (x-3)}$$ Rewrite the fraction with the monomional terms first and the binominal terms last.

cancel the common binominal factors $\frac{(x-3)}{(x-3)}$ and $\frac{(x+2)}{(x+2)}$

$\frac{x}{5}$ does not reduce

$$= \frac{x\ \cancel{(x-3)}(x+2)\ \cancel{(x+2)}}{5\ \cancel{(x+2)}(x+1)\ \cancel{(x-3)}} = \frac{x\ (x+2)}{5\ (x+1)}$$

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Example 8

$\dfrac{2x^2-10x}{21x} \div \dfrac{x^2-25}{14x^2}$ Invert the second fraction and multiply

$= \dfrac{2x^2-10x}{21x} \bullet \dfrac{14x^2}{x^2-25}$

$= \dfrac{2x(x-5)}{21x} \bullet \dfrac{14x^2}{(x+5)(x-5)}$ factor each numerator
factor each denominator

$= \dfrac{28\ x^3\ (x-5)}{21\ x\ (x+5)(x-5)}$ Rewrite the fraction with the monomional terms first and the binominal terms last.

cancel the common binominal factors $\dfrac{(x-5)}{(x-5)}$

and reduce $\dfrac{28x^3}{21x}$ to $\dfrac{4x^2}{3}$ using the quoient rule

$= \dfrac{\overset{7}{\cancel{28}}\ \overset{x^2}{\cancel{x^3}}\ \cancel{(x-5)}}{\underset{3}{\cancel{21}}\ \cancel{x}\ (x+5)\ \cancel{(x-5)}} = \dfrac{7x^2}{3(x+5)}$

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Example 9

$\dfrac{3x+6}{x^2-9} \div \dfrac{5x+10}{x-3}$ Invert the second fraction and multiply

$= \dfrac{3x+6}{x^2-9} \bullet \dfrac{x-3}{5x+10}$

$= \dfrac{3(x+2)}{(x+3)(x-3)} \bullet \dfrac{(x-3)}{5(x+2)}$ factor each numerator
factor each denominator

$= \dfrac{3\ (x+2)\ (x-3)}{5\ (x+3)\ (x-3)\ (x+2)}$ Rewrite the fraction with the monomional terms first and the binominal terms last.

cancel the common binominal factors $\dfrac{(x+2)}{(x+2)}\ \dfrac{(x-3)}{(x-3)}$

the $\dfrac{3}{5}$ does not reduce

$= \dfrac{3\ \cancel{(x+2)}\ \cancel{(x-3)}}{5\ (x+3)\ \cancel{(x-3)}\ \cancel{(x+2)}} = \dfrac{3}{5(x+3)}$

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