

Section 8 – 1:

Reducing Rational Expressions

Rational Expressions

A polynomial is an expression containing the sum or difference of two or more terms. Each of the terms is separated by a + or – sign.

A **Rational Expression** is a **fraction** with a polynomial in the numerator and denominator.

$$\frac{2x^2 - 10x}{4x}$$

$$\frac{8x^2 - 6x}{x - 8}$$

Reducing a Fraction to Lowest Terms

We reduce a fraction by **completely factoring** the numerator and denominator. After we have factored the numerator and denominator into **products** we can **cancel out the common factors**.

$$\begin{aligned} &\text{Reduce } \frac{15}{6} \\ &= \frac{3 \cdot 5}{3 \cdot 2} \text{ factor} \\ &\quad \text{cancel common factors} \\ &= \frac{3 \cdot 5}{3 \cdot 2} = \frac{5}{2} \end{aligned}$$

You cannot cancel parts of a sum.

We know that $\frac{15}{6}$ reduces to $\frac{5}{2}$

$$\frac{15}{6} = \frac{12+3}{6} \text{ so } \frac{12+3}{6} \text{ must reduce to } \frac{5}{2}$$

Try to reduce $\frac{12+3}{6}$
by canceling the 12 and 6

$$\frac{12^6+3}{6^2} = \frac{6+3}{2} = \frac{9}{2}$$

which is not $\frac{5}{2}$

You cannot cancel the 12 in the numerator with the 6 in the denominator.
It will not result in a true statement.

Try to reduce $\frac{12+3}{6}$
by canceling the 3 and 6

$$\frac{12+3^1}{6^2} = \frac{12+1}{2} = \frac{13}{2}$$

which is not $\frac{5}{2}$

You cannot cancel the 3 in the numerator with the 6 in the denominator.
It will not result in a true statement.

Reducing Rational Expressions

You cannot cancel part of an expression that has addition or subtraction.

You CANNOT cancel any part of a polynomial expression in the numerator with any part of a polynomial expression in the denominator.

Example 1

$$\frac{5x+6}{3}$$

You **CANNOT** cancel
the 6 and the 3

Example 2

$$\frac{4x^2+2x}{6x}$$

You **CANNOT** cancel
the $4x^2$ and the $6x$
or the $2x$ and the $6x$

When you factor the polynomial expression into separate factors you CAN CANCEL COMMON FACTORS.

Example 4

$$\frac{(x-1)(x-2)}{(x-6)(x-1)}$$

the $(x-1)$, $(x-2)$ and the $(x-6)$
are each separate factors or products so
you can cancel the $(x-1)$ in the numerator
with the $(x-1)$ in the denominator

Reduce

$$\frac{(x-1)(x-2)}{(x-6)(x-1)}$$

the $\frac{(x-1)}{(x-1)}$ can be canceled

$$= \frac{\cancel{(x-1)}(x-2)}{(x-6)\cancel{(x-1)}} = \frac{(x-2)}{(x-6)}$$

You **CANNOT** cancel
part of a binomial term.
The 2 in the $(x-2)$ term in
the numerator of the answer
CANNOT be canceled with the 6
in the $(x-6)$ term in the denominator.

Example 5

$$\frac{(x-2)(x+3)}{3(x+3)}$$

the $(x-2)$, $(x+3)$ and the 3
are each separate factors or products so
you can cancel the $(x+3)$ in the numerator
with the $(x+3)$ in the denominator

Reduce

$$\frac{5x(x+3)}{5(x+3)(x-1)}$$

the $\frac{(x+3)}{(x+3)}$ and the $\frac{5}{5}$ can be canceled

$$= \frac{5^1 x \cancel{(x+3)}}{5^1 \cancel{(x+3)} (x-1)} = \frac{x}{(x-1)}$$

You **CANNOT** cancel
part of a binomial term.
The x term in the numerator
of the answer **CANNOT**
be canceled with the x in the $(x-1)$
term in the denominator.

Reducing Rational Expressions

We reduce a **Rational Expression completely by factoring** the polynomial expressions in the numerator and denominator and then canceling out all the common factors.

1. **Completely factor** the numerator and denominator.
2. Write the fraction with the **monomial terms first** and the **binomial terms last**.
3. Cancel out the common binomial factors and use the quotient rule to reduce the monomial terms.
4. Write the remaining factors in fraction form.

Example 1

$$\frac{10x^2 - 15x}{4x^2 - 9} \quad \begin{array}{l} \text{factor the numerator (GCF)} \\ \text{factor the denominator (Diff. of Square)} \end{array}$$

$$= \frac{5x(2x - 3)}{(2x - 3)(2x + 3)}$$

cancel the common binomial factors $\frac{(2x - 3)}{(2x - 3)}$

$$= \frac{5x\cancel{(2x - 3)}}{\cancel{(2x - 3)}(2x + 3)} = \frac{5x}{(2x + 3)}$$

Example 2

$$\frac{4x^2 - 8x}{2x^3 - 4x^2} \quad \begin{array}{l} \text{factor the numerator (GCF)} \\ \text{factor the denominator (GCF)} \end{array}$$

$$= \frac{4x(x - 2)}{2x^2(x - 2)}$$

cancel the common binomial factors $\frac{(x - 2)}{(x - 2)}$

and reduce $\frac{4x}{2x^2}$ to $\frac{2}{x}$ using the quotient rule

$$= \frac{4^2 \cancel{x} \cancel{(x - 2)}}{2^1 \cancel{x^2} \cancel{(x - 2)}} = \frac{2}{x}$$

Example 3

$$\frac{x^2 - x - 6}{x^2 - 6x + 9} \quad \begin{array}{l} \text{factor the numerator (trinomial)} \\ \text{factor the denominator (trinomial)} \end{array}$$

$$= \frac{(x-3)(x+2)}{(x-3)(x-3)}$$

cancel the common binominal factors $\frac{(x-3)}{(x-3)}$

$$= \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x-3)} = \frac{(x+2)}{(x-3)}$$

Example 4

$$\frac{3x^2y^2 - 9xy^2}{12x^2y - 6xy} \quad \begin{array}{l} \text{factor the numerator (GCF)} \\ \text{factor the denominator (GCF)} \end{array}$$

$$= \frac{3xy^2(x-3)}{6xy(2x-1)}$$

There are no common binominal factors

reduce $\frac{3xy^2}{6xy}$ to get $\frac{y}{2}$ using the quotient rule

$$= \frac{\overset{y}{3^1} \cancel{x} \cancel{y^2} (x-3)}{\underset{2}{6^2} \cancel{x} \cancel{y} (2x-1)} = \frac{y(x-3)}{2(2x-1)}$$

Example 5

$$\frac{2x^3 - 8x}{4x^2 - 12x - 16} \quad \begin{array}{l} \text{factor the numerator (GCF)} \\ \text{factor the denominator (GCF)} \end{array}$$

$$= \frac{2x(x-4)}{4(x^2 - 3x - 4)} \quad \text{factor the denominator (trinomial)}$$

$$= \frac{2x(x-4)}{4(x+1)(x-4)}$$

cancel the common binomial factors $\frac{(x-4)}{(x-4)}$

reduce $\frac{2x}{4}$ to get $\frac{x}{2}$ using the quotient rule

$$= \frac{2^1 x \cancel{(x-4)}}{4^2 (x+1) \cancel{(x-4)}} = \frac{x}{2(x+1)}$$

Example 6

$$\frac{2x^3 - 8x}{8x^2 + 4x - 12} \quad \begin{array}{l} \text{factor the numerator (GCF)} \\ \text{factor the denominator (GCF)} \end{array}$$

$$= \frac{2x(4x^2 - 9)}{4(2x^2 + x - 3)} \quad \begin{array}{l} \text{factor the numerator (Diff. of 2 Perfect Square)} \\ \text{factor the denominator (Hard Trinomial)} \end{array}$$

$$= \frac{2x(2x-3)(2x+3)}{4(2x+3)(x-1)}$$

cancel the common binomial factors $\frac{(2x+3)}{(2x+3)}$

reduce $\frac{2x}{4}$ to get $\frac{x}{2}$ using the quotient rule

$$= \frac{2^1 x (2x-3) \cancel{(2x+3)}}{4^2 \cancel{(2x+3)} (x-1)} = \frac{x(2x-3)}{2(x-1)}$$

The Difference of 2 Perfect Squares is normally written with a **positive** x^2 term written first and a negative constant term written second

$$x^2 - 9$$

$$4x^2 - 25$$

$$49x^2 - 1$$

If the Difference of 2 Perfect Squares is written with the a **positive** constant term written first and a negative x^2 term written second then we will factor out a -1 first to change the form to the more common format.

$$9 - x^2 = -(x^2 - 9)$$

$$25 - 4x^2 = -(4x^2 - 25)$$

$$1 - 49x^2 = -(49x^2 - 1)$$

Example 7

$$\frac{5x - 10}{4 - x^2} \text{ factor out a } -1$$

$$\frac{5x - 10}{-(x^2 - 4)} \text{ factor the numerator (GCF) factor the denominator (Diff. of Squares)}$$

$$= \frac{5 \cancel{(x - 2)}}{-(x + 2) \cancel{(x - 2)}} \text{ cancel the common factors}$$

$$= \frac{5}{-(x + 2)} \text{ or } \frac{-5}{(x + 2)}$$

Note: In past chapters, if we had a fraction as an answer with a negative sign in the denominator such as $\frac{5}{-2}$ we would rewrite it as $\frac{-5}{2}$. We will not require that practice in this chapter but you could if you

desired. That would change the answer to Example 8 from $\frac{5}{-(x + 2)}$ to $\frac{-5}{(x + 2)}$

Example 8

$$\frac{16 - x^2}{3x^2 - 12x} \quad \text{factor out a } -1$$

$$\frac{-(x^2 - 16)}{3x^2 - 12x} \quad \begin{array}{l} \text{factor the numerator (Diff. of Squares)} \\ \text{factor the denominator (GCF)} \end{array}$$

$$= \frac{-\cancel{(x-4)}(x+4)}{3x\cancel{(x-4)}} \quad \text{cancel the common factors}$$

$$= \frac{-(x+4)}{3x}$$

Note: The **denominator of a fraction cannot have a value of zero**. When we have a polynomial in the denominator any value of the variable that would make the denominator zero is not allowed. This restriction will not be explored in the first part of the chapter. We will revisit it in the last section.

Example 1

$$\frac{2x^2 - 10x}{4x} ; x \neq 0$$

$$4x \neq 0$$

$$x \neq 0$$

Example 2

$$\frac{8x^2 - 6x}{x + 8} ; x \neq -8$$

$$x + 8 \neq 0$$

$$x \neq -8$$

Example 3

$$\frac{5x}{2x - 3} ; x \neq \frac{3}{2}$$

$$2x - 3 \neq 0$$

$$2x \neq 3$$

$$x \neq \frac{3}{2}$$