Tool 1

Greatest Common Factor (GCF)

This is a very important tool. You must try to factor out the GCF first in every problem. Some problems do not have a GCF but many do. When you factor out the GCF you may recognize the expression inside the parentheses as a term that can be factored further by one of the other types of factoring tools.

Example 1

Factor:

\[5x^2y + 20xy - 5y\]

(the GCF is 5y)

\[\frac{5x^2y}{5y} + \frac{20xy}{5y} - \frac{5y}{5y}\]

= \(5y(x^2 + 4x - 1)\)

Example 2

Factor:

\[10x^3 - 20x^2 + 50x\]

(the GCF is 10x)

\[\frac{10x^3}{10x} - \frac{20x^2}{10x} + \frac{50x}{10x}\]

= \(10x(x^2 - 2x + 5)\)

Tool 2

Factor By Grouping Method

Factoring by Grouping requires four terms. You group the first two terms together and the last two terms together and then factor out the GCF in the first group and factor out the GCF in the second group. The final step requires that you take out the common binomial in each term. You will then have the final factored form which is the product of two binomials.

Example 3

Factor:

\[6x^2 - 4x - 15x + 10\]

\[2x (3x - 2) - 5 (3x - 2)\]

\[(3x - 2) ( 2x - 5)\]

Example 4

Factor:

\[20x^2 + 12x - 5x - 3\]

\[4x (5x + 3) - 1 (5x + 3)\]

\[(5x + 3) ( 4x - 1)\]
Tool 3A

Factoring The **Difference of Two Perfect Squares**

This tool requires *Two Terms that are Perfect Squares separated by a – sign.*

The factors of The Difference of Two Perfect Squares are **the product of the sum and difference of the factors** that make up each of the perfect squares.

**Example 5**

Factor:

\[4x^2 - 25\]

\[4x^2 = 2x \cdot 2x\] and \[25 = 5 \cdot 5\]

\[= (2x + 5)(2x - 5)\]

**Example 6**

Factor:

\[9x^2 - 25y^2\]

\[9x^2 = 3x \cdot 3x\] and \[5y^2 = 5y \cdot 5y\]

\[= (3x + 5y)(3x - 5y)\]

Tool 3B

The **SUM of Two Perfect Squares**

This tool requires *Two Terms that are Perfect Squares separated by a + sign.**

The **Sum of Two Perfect Squares** Does **NOT Factor (DNF)**

**Example 7**

\[9x^2 + 16\]

Does not factor

DNF

**Example 8**

\[4x^2 + 25\]

Does not factor

DNF
Tool 4A

Factoring The Difference of Two Perfect Cubes

This tool requires Two Terms that are Perfect Cubes separated by a – sign.

Factor: \( a^3 - b^3 \)

\[ = (a - b) \left( a^2 + ab + b^2 \right) \]

\((a-b)\) is found by \(\text{perfect cube root of the first term \ keep the same sign \ perfect cube root of the last term}\)

\((a-b) \left( a^2 + ab + b^2 \right)\) is found by

\((\text{First} - \text{Last})\) \(\left( \text{First}^2 \ \text{change sign} \ \text{First} \cdot \text{Last} \ \text{always} + \ \text{Last}^2 \right)\)

**Example 9**

Factor: \(8x^3 - 125\)

\[= (2x)^3 - 5^3\]

\[= (2x - 5) \left( 4x^2 + 10x + 25 \right)\]

\(F \ L \quad F^2 \text{ cs } F \cdot L + L^2\)

**Example 10**

Factor: \(64x^3 - 27\)

\[= (4x)^3 - 3^3\]

\[= (4x - 3) \left( 4x^2 + 12x + 9 \right)\]

\(F \ L \quad F^2 \text{ cs } F \cdot L + L^2\)
Tool 4B

The SUM of Two Perfect Cubes

This tool requires Two Terms that are Perfect Cubes separated by a + sign.

Factoring The Sum of Two Perfect Cubes

Factor: \( a^3 + b^3 \)

\[ = (a + b) \left( a^2 - ab + b^2 \right) \]

\((a+b)\) is found by \(\begin{array}{c} \text{perfect cube root} \\ \text{of the first term} \end{array} \begin{array}{c} \text{keep the} \\ \text{same sign} \end{array} \begin{array}{c} \text{perfect cube root} \\ \text{of the last term} \end{array} \)

\((a + b) \left( a^2 - ab + b^2 \right)\) is found by

First - Last \(\begin{array}{c} \text{First}^2 \\ \text{change sign} \end{array} \begin{array}{c} \text{First} \bullet \text{Last} \\ \text{always + Last}^2 \end{array} \)

**Example 11**

Factor: \( 27x^3 - 8 \)

\[ = (3x)^3 - 2^3 \]

\[
(3x + 2) \left( 9x^2 - 6x + 4 \right) \]

F L F² cs F•L + L²

**Example 12**

Factor: \( 125x^3 + 64 \)

\[ = (5x)^3 + 4^3 \]

\[
(5x + 4) \left( 25x^2 - 20x + 16 \right) \]

F L F² cs F•L + L²
Factoring Easy Trinomials
that have $1x^2$ as the first term and ends with a positive number (C is positive).

Factor
$$1x^2 + Bx + C$$
into $(x + D)(x + E)$

$D$ and $E$ must MULTIPLY to $+C$
and ADD to $±B$

**Example 13**
Factor $x^2 - 9x + 20$
into $(x + D)(x + E)$

$$
\begin{array}{c}
1 \cdot 20 \\
2 \cdot 10 \\
4 \cdot 5 \\
\end{array}
$$

$x^2 - 9x + 20$

we need two numbers
$D$ and $E$ that
multiply to $+20$
and add to $-9$

$-4$ and $-5$ work

**Answer:** $(x - 4)(x - 5)$

**Example 14**
Factor $x^2 + 9x + 18$
into $(x + D)(x + E)$

$$
\begin{array}{c}
1 \cdot 18 \\
2 \cdot 9 \\
3 \cdot 6 \\
\end{array}
$$

$x^2 + 9x + 18$

we need two numbers
$D$ and $E$ that
multiply to $+18$
and add to $+9$

$+3$ and $+6$ work

**Answer:** $(x + 3)(x + 6)$

**Example 15**
Factor $x^2 - 7x + 12$
into $(x + D)(x + E)$

$$
\begin{array}{c}
1 \cdot 12 \\
2 \cdot 6 \\
3 \cdot 4 \\
\end{array}
$$

$x^2 - 7x + 12$

we need two numbers
$D$ and $E$ that
multiply to $+12$
and add to $-7$

$-3$ and $-4$ work

**Answer:** $(x - 3)(x - 4)$
Tool 5B

Factoring Easy Trinomials
that have $1x^2$ as the first term and
ends with a negative number (C is negative).

Factor

$$1x^2 + Bx - C$$

into $(x + D)(x + E)$

$D$ and $E$ must Multiply to $-C$
and SUBTRACT to $\pm B$

Example 16

Factor $x^2 - x - 20$
into $(x + D)(x + E)$

$1\cdot20$
$2\cdot10$
$4\cdot5$

$x^2 - x - 20$
we need two numbers
$D$ and $E$ that
multiply to $-20$
and subtract to $-1$
$-5$ and $4$ work

Answer: $(x-5)(x+4)$

Example 17

Factor $x^2 + 3x - 18$
into $(x + D)(x + E)$

$1\cdot18$
$2\cdot9$
$3\cdot6$

$x^2 + 3x - 18$
we need two numbers
$D$ and $E$ that
multiply to $-18$
and subtract to $+3$
$+6$ and $-3$ work

Answer: $(x+6)(x-3)$

Example 18

Factor $x^2 - 4x - 12$
into $(x + D)(x + E)$

$1\cdot12$
$2\cdot6$
$3\cdot4$

$x^2 - 4x - 12$
we need two numbers
$D$ and $E$ that
multiply to $-12$
and subtract to $-4$
$+6$ and $-2$ work

Answer: $(x+6)(x-2)$
Tool 6A

Factoring Hard Trinomials like $Ax^2 \pm Bx \pm C$ where $A > 1$
that END with a POSITIVE number (C is POSITIVE)
by
the Creating an Easy Trinomial Method

Example 19

Factor: $2x^2 - 9x + 10$

Step 1: The **GCF must be taken out first (if there is one)** before factoring the hard trinomial.

Step 2: Create an Easy Trinomial by moving the coefficient of the $2x^2$ term to the end of the trinomial and multiplying the 2 and the 10

$$2x^2 - 9x + 10 \cdot 2$$

to get the easy trinomial

$$x^2 - 9x + 20$$

Step 3: Factor the easy trinomial by finding the 2 numbers that multiply to +20 and add to –9

–5 and –4

$(x - 5)(x - 4)$

Step 4: In step 1 you multiplied the constant 10 by the 2 that was the coefficient of the $2x^2$ term.
Now divide BOTH of the constants in $(x - 5)(x - 4)$ by 2

$$\left( \frac{x - 5}{2} \right) \left( \frac{x - 4}{2} \right)$$

reduce each fraction

$$\left( \frac{x - 5}{2} \right)(x - 2)$$

Step 5: "glide" the denominator of each fraction (if there is one) to the front of the x term

$$\left( \frac{2x - 5}{2} \right)(x - 2)$$

Answer: $(2x - 5)(x - 2)$
Tool 6B

Factoring Hard Trinomials like $Ax^2 \pm Bx \pm C$ where $A > 1$ that END with a NEGATIVE number ($C$ is NEGATIVE)

by

the Creating an Easy Trinomial Method

Example 20

Factor: $6x^2 - x - 1$

Step 1: The GCF must be taken out first (if there is one) before factoring the hard trinomial.

Step 2: Create an Easy Trinomial by moving the coefficient of the $6\,x^2$ term to the end of the trinomial and multiplying the 6 and the −1

$$6x^2 - x - 1 \cdot 6$$

to get the easy trinomial

$$x^2 - x - 6$$

Step 3: Factor the easy trinomial by finding the 2 numbers that multiply to −6 and subtract to −1

−3 and + 2

$$(x - 3) \, (x + 2)$$

Step 4: In step 1 you multiplied the constant −1 by the 6 that was in front of the $x^2$ term. Now divide BOTH of the constants in $(x - 3) \, (x + 2)$ by 6

$$\left( \frac{x - 3}{6} \right) \left( \frac{x + 2}{6} \right)$$

reduce each fraction

$$\left( \frac{x - 1}{2} \right) \left( \frac{x + 1}{3} \right)$$

Step 5: "glide" the denominator of each fraction (if there is one) to the front of the x term

$$\left( \frac{2}{x - \frac{1}{2}} \right) \left( \frac{3}{x + \frac{1}{3}} \right)$$

Answer: $(2x - 1) \, (3x + 1)$
Tool 7A

Factoring Hard Trinomials like $Ax^2 \pm Bx \pm C$ where $A > 1$ that END with a POSITIVE number (C is POSITIVE) by

the AC – Factoring By Grouping Method

Example 21

Factor: $3x^2 + 7x + 2$

Step 1: Multiply the two outer terms $3x^2$ and the 2 to get $+6x^2$

$3x^2 + 7x + 2$

Step 2: Find 2 terms that multiply to $+6x^2$ and add to $+7x$ $+6x$ and $+1x$

Step 3: Replace the $+7x$ in $3x^2 + 7x + 2$ with $+6x + 1x$

$3x^2 + 6x + 1x + 2$

to get $3x^2 + 6x + 1x + 2$

Step 4: $3x^2 + 6x + 1x + 2$

Factor a GCF of $3x$ out of the first two terms and Factor a GCF of 1 out of the last two terms

$3x(x + 2) + 1(x + 2)$

Factor a GCF of $(x + 2)$ out of the two terms

Answer: $(x + 2)(3x + 1)$
Factoring Hard Trinomials like $Ax^2 \pm Bx \pm C$ where $A > 1$

that END with a NEGATIVE number ($C$ is NEGATIVE)

by

the AC – Factoring By Grouping Method

Example 22

Factor: $12x^2 + 5x - 2$

Step 1: Multiply the two outer terms
$12x^2$ and the $-2$ to get $-24x^2$

Step 2: Find 2 terms that multiply to $-24x^2$ and subtract to $+5x$
$+8x$ and $-3x$

Replace the $+5x$ in

$12x^2 + 5x - 2$ with

$+8x - 3x$

$12x^2 \downarrow \downarrow \downarrow \downarrow - 2$

to get

$12x^2 + 8x - 3x - 2$

Step 4:

$12x^2 + 8x - 3x - 2$

Factor a GCF of $4x$ out of the first two terms and
Factor a GCF of $-1$ out of the last two terms

$4x(3x + 2) - 1(3x + 2)$

Factor a GCF of $(3x + 2)$ out of the two terms

Answer: $(3x + 2)(4x - 1)$