

Tool 1

Greatest Common Factor (GCF)

This is a very important tool. **You must try to factor out the GCF first in every problem.** Some problems do not have a GCF but many do. When you factor out the GCF you may recognize the **expression inside the parentheses** as a term that can be **factored further** by one of the other types of factoring tools.

Example 1

Factor:

$$5x^2y + 20xy - 5y$$

(the GCF is 5y)

$$\frac{5x^2y}{5y} + \frac{20xy}{5y} - \frac{5y}{5y}$$

$$= 5y(x^2 + 4x - 1)$$

Example 2

Factor:

$$10x^3 - 20x^2 + 50x$$

(the GCF is 10x)

$$\frac{10x^3}{10x} - \frac{20x^2}{10x} + \frac{50x}{10x}$$

$$= 10x(x^2 - 2x + 5)$$

Tool 2

Factor By Grouping Method

Factoring by Grouping requires **four terms**. You **group the first two terms together** and **the last two terms together** and then **factor out the GCF in the first group** and **factor out the GCF in the second group**. The final step requires that you take out the **common binomial** in each term. You will then have the final factored form which is the product of two binomials.

Example 3

Factor:

$$6x^2 - 4x - 15x + 10$$

$$2x(3x - 2) - 5(3x - 2)$$

$$(3x - 2)(2x - 5)$$

Example 4

Factor:

$$20x^2 + 12x - 5x - 3$$

$$4x(5x + 3) - 1(5x + 3)$$

$$(5x + 3)(4x - 1)$$

Tool 3A

Factoring The **Difference** of **Two Perfect Squares**

This tool requires **Two Terms that are Perfect Squares** separated by a **– sign**.

The factors of The Difference of Two Perfect Squares are **the product of the sum and difference of the factors** that make up each of the perfect squares.

Example 5

Factor:

$$4x^2 - 25$$

$$4x^2 = 2x \bullet 2x \text{ and } 25 = 5 \bullet 5$$

$$= (2x + 5)(2x - 5)$$

Example 6

Factor:

$$9x^2 - 25y^2$$

$$9x^2 = 3x \bullet 3x \text{ and } 25y^2 = 5y \bullet 5y$$

$$= (3x + 5y)(3x - 5y)$$

Tool 3B

The **SUM** of **Two Perfect Squares**

This tool requires **Two Terms that are Perfect Squares** separated by a **+ sign**.

The **Sum of Two Perfect Squares** Does **NOT** Factor (DNF)

Example 7

$$9x^2 + 16$$

Does not factor

DNF

Example 8

$$4x^2 + 25$$

Does not factor

DNF

Tool 4A

Factoring The Difference of Two Perfect Cubes

This tool requires **Two Terms that are Perfect Cubes** separated by a **– sign**.

$$\begin{aligned} \text{Factor: } & a^3 - b^3 \\ & = (a - b) (a^2 + ab + b^2) \end{aligned}$$

$$(a - b) \text{ is found by } \left(\begin{array}{ccc} \text{perfect cube root} & \text{keep the} & \text{perfect cube root} \\ \text{of the first term} & \text{same sign} & \text{of the last term} \end{array} \right)$$

$$\begin{aligned} (a - b) (a^2 + ab + b^2) \text{ is found by} \\ \text{(First - Last) (First}^2 \text{ change sign First} \bullet \text{Last always + Last}^2) \end{aligned}$$

Example 9

$$\text{Factor: } 8x^3 - 125$$

$$= (2x)^3 - 5^3$$

$$= (2x - 5) (4x^2 + 10x + 25)$$

$$\text{F L } \quad \text{F}^2 \text{ cs F} \bullet \text{L} + \text{L}^2$$

Example 10

$$\text{Factor: } 64x^3 - 27$$

$$= (4x)^3 - 3^3$$

$$(4x - 3) (4x^2 + 12x + 9)$$

$$\text{F L } \quad \text{F}^2 \text{ cs F} \bullet \text{L} + \text{L}^2$$

Tool 4B

The **SUM** of Two Perfect Cubes

This tool requires **Two Terms that are Perfect Cubes** separated by a + sign.

Factoring The Sum of Two Perfect Cubes

$$\begin{aligned} \text{Factor: } & a^3 + b^3 \\ & = (a + b) (a^2 - ab + b^2) \end{aligned}$$

$$(a + b) \text{ is found by } \left(\begin{array}{ccc} \text{perfect cube root} & \text{keep the} & \text{perfect cube root} \\ \text{of the first term} & \text{same sign} & \text{of the last term} \end{array} \right)$$

$$(a + b) (a^2 - ab + b^2) \text{ is found by}$$

$$(\text{First} - \text{Last}) (\text{First}^2 \text{ change sign } \text{First} \bullet \text{Last} \text{ always} + \text{Last}^2)$$

Example 11

$$\text{Factor: } 27x^3 - 8$$

$$= (3x)^3 - 2^3$$

$$(3x + 2) (9x^2 - 6x + 4)$$

$$\text{F } L \quad \text{F}^2 \text{ cs } \text{F} \bullet \text{L} + \text{L}^2$$

Example 12

$$\text{Factor: } 125x^3 + 64$$

$$= (5x)^3 + 4^3$$

$$(5x + 4) (25x^2 - 20x + 16)$$

$$\text{F } L \quad \text{F}^2 \text{ cs } \text{F} \bullet \text{L} + \text{L}^2$$

Tool 5A

Factoring Easy Trinomials

that have $1x^2$ as the first term and
ends **with a positive number (C is positive)**.

Factor

$$1x^2 \pm Bx + C$$

$$\text{into } (x \pm D) (x \pm E)$$

D and E must MULTIPLY to + C
and **ADD to ± B**

Example 13

Factor $x^2 - 9x + 20$
into $(x \pm D) (x \pm E)$

$$1 \cdot 20$$

$$2 \cdot 10$$

$$4 \cdot 5$$

$$x^2 - 9x + 20$$

we need two numbers

D and E that
multiply to + 20
and **add to - 9**

- 4 and - 5 work

Answer: $(x - 4) (x - 5)$

Example 14

Factor $x^2 + 9x + 18$
into $(x \pm D) (x \pm E)$

$$1 \cdot 18$$

$$2 \cdot 9$$

$$3 \cdot 6$$

$$x^2 + 9x + 18$$

we need two numbers

D and E that
multiply to + 18
and **add to + 9**

+ 3 and + 6 work

Answer: $(x + 3) (x + 6)$

Example 15

Factor $x^2 - 7x + 12$
into $(x \pm D) (x \pm E)$

$$1 \cdot 12$$

$$2 \cdot 6$$

$$3 \cdot 4$$

$$x^2 - 7x + 12$$

we need two numbers

D and E that
multiply to + 12
and **add to - 7**

- 3 and - 4 work

Answer: $(x - 3) (x - 4)$

Tool 5B

Factoring Easy Trinomials

that have $1x^2$ as the first term and
ends with a negative number (C is negative).

Factor

$$1x^2 \pm Bx - C$$

$$\text{into } (x \pm D) (x \pm E)$$

D and **E** must **Multiply** to $-C$
and **SUBTRACT** to $\pm B$

Example 16

Factor $x^2 - x - 20$
into $(x \pm D) (x \pm E)$

$$1 \cdot 20$$

$$2 \cdot 10$$

$$4 \cdot 5$$

$$x^2 - x - 20$$

we need two numbers

D and **E** that

multiply to -20

and **subtract to -1**

-5 and **4** work

Answer: $(x - 5)(x + 4)$

Example 17

Factor $x^2 + 3x - 18$
into $(x \pm D) (x \pm E)$

$$1 \cdot 18$$

$$2 \cdot 9$$

$$3 \cdot 6$$

$$x^2 + 3x - 18$$

we need two numbers

D and **E** that

multiply to -18

and **subtract to $+3$**

$+6$ and **-3** work

Answer: $(x + 6)(x - 3)$

Example 18

Factor $x^2 - 4x - 12$
into $(x \pm D) (x \pm E)$

$$1 \cdot 12$$

$$2 \cdot 6$$

$$3 \cdot 4$$

$$x^2 - 4x - 12$$

we need two numbers

D and **E** that

multiply to -12

and **subtract to -4**

$+6$ and **-2** work

Answer: $(x + 6)(x - 2)$

Tool 6A

Factoring Hard Trinomials like $Ax^2 \pm Bx \pm C$ where $A > 1$
that **END with a POSITIVE number (C is POSITIVE)**
by
the Creating an Easy Trinomial Method

Example 19

$$\text{Factor: } 2x^2 - 9x + 10$$

Step 1: The **GCF must be taken out first (if there is one)** before factoring the hard trinomial.

Step 2: Create an Easy Trinomial by moving the coefficient of the $2x^2$ term to the end of the trinomial and multiplying the **2** and the 10

$$\begin{array}{c} \boxed{} \\ \downarrow \\ 2x^2 - 9x + 10 \bullet 2 \end{array}$$

to get the easy trinomial

$$x^2 - 9x + 20$$

Step 3: Factor the easy trinomial by finding the 2 numbers that **multiply to + 20 and add to -9**

$$- 5 \text{ and } - 4$$

$$(x - 5)(x - 4)$$

Step 4: In step 1 you multiplied the constant 10 by the **2** that was the coefficient of the $2x^2$ term. Now **divide BOTH of the constants** in $(x - 5)(x - 4)$ by **2**

$$\left(x - \frac{5}{2}\right)\left(x - \frac{4}{2}\right)$$

reduce each fraction

$$\left(x - \frac{5}{2}\right)(x - 2)$$

Step 5: "glide" the denominator of each fraction (if there is one) to the front of the x term

$$\begin{array}{c} \left(2x - \frac{5}{2}\right)(x - 2) \\ \uparrow \quad \downarrow \\ \boxed{\phantom{2x - \frac{5}{2}}} \end{array}$$

$$\text{Answer: } (2x - 5)(x - 2)$$

Tool 6B

Factoring Hard Trinomials like $Ax^2 \pm Bx \pm C$ where $A > 1$
that **END with a NEGATIVE number (C is NEGATIVE)**
by
the Creating an Easy Trinomial Method

Example 20

Factor: $6x^2 - x - 1$

Step 1: The **GCF must be taken out first (if there is one)** before factoring the hard trinomial.

Step 2: Create an Easy Trinomial by moving the coefficient of the $6x^2$ term to the end of the trinomial and multiplying the **6** and the -1

$$6x^2 - x - 1 \cdot \mathbf{6}$$

to get the easy trinomial

$$x^2 - x - 6$$

Step 3: Factor the easy trinomial by finding the 2 numbers that **multiply to -6 and subtract to -1**

$$-3 \text{ and } +2$$

$$(x - 3)(x + 2)$$

Step 4: In step 1 you multiplied the constant -1 by the **6** that was in front of the x^2 term.
Now **divide BOTH of the constants** in $(x - 3)(x + 2)$ by **6**

$$\left(x - \frac{3}{6}\right)\left(x + \frac{2}{6}\right)$$

reduce each fraction

$$\left(x - \frac{1}{2}\right)\left(x + \frac{1}{3}\right)$$

Step 5: "glide" the denominator of each fraction (if there is one) to the front of the x term

$$\left(\mathbf{2}x - \frac{1}{2}\right)\left(\mathbf{3}x + \frac{1}{3}\right)$$

Answer: $(2x - 1)(3x + 1)$

Tool 7A

Factoring Hard Trinomials like $Ax^2 \pm Bx \pm C$ where $A > 1$
that **END with a POSITIVE number (C is POSITIVE)**
by
the AC – Factoring By Grouping Method

Example 21

Factor: $3x^2 + 7x + 2$

Step 1: Multiply the two outer terms
 $3x^2$ and the 2 to get $+6x^2$

$$\begin{array}{c} \overbrace{\hspace{1.5cm}} \\ 3x^2 + 7x + 2 \end{array}$$

Step 2: Find 2 terms that **multiply to $+6x^2$** and **add to $+7x$**
 $+6x$ and $+1x$

Step 3: Replace the $+7x$ in
 $3x^2 + 7x + 2$ with

$$\begin{array}{c} +6x + 1x \\ \downarrow \quad \downarrow \\ 3x^2 \quad \underline{\quad} \quad \underline{\quad} \quad + 2 \end{array}$$

to get

$$3x^2 + 6x + 1x + 2$$

Step 4: $3x^2 + 6x + 1x + 2$

**Factor a GCF of $3x$ out of the first two terms and
Factor a GCF of 1 out of the last two terms**

$$3x(x + 2) + 1(x + 2)$$

Factor a GCF of $(x + 2)$ out of the two terms

Answer: $(x + 2)(3x + 1)$

Tool 7B

Factoring Hard Trinomials like $Ax^2 \pm Bx \pm C$ where $A > 1$
that **END with a NEGATIVE number (C is NEGATIVE)**
by

the AC – Factoring By Grouping Method

Example 22

Factor: $12x^2 + 5x - 2$

Step 1: Multiply the two outer terms

$12x^2$ and the -2 to get $-24x^2$

Step 2: Find 2 terms that **multiply to $-24x^2$** and **subtract to $+5x$**
 $+8x$ and $-3x$

Replace the **$+5x$** in

$12x^2 + 5x - 2$ with

$$\begin{array}{ccc} & +8x - & 3x \\ & \downarrow & \downarrow \\ 12x^2 & \underline{\quad} & \underline{\quad} - 2 \end{array}$$

to get

$$12x^2 + 8x - 3x - 2$$

Step 4: $12x^2 + 8x - 3x - 2$

Factor a GCF of $4x$ out of the first two terms and
Factor a GCF of -1 out of the last two terms

$$4x(3x + 2) - 1(3x + 2)$$

Factor a GCF of $(3x + 2)$ out of the two terms

Answer: $(3x + 2)(4x - 1)$