Section 7 – 5: Solving 2nd Degree Equations

First Degree Equations

First degree equations contain **variable terms to the first power** and constants.

$2x-6=14$ $\qquad\qquad$ $2x+3=4x-15$

First Degree Equations are solved by getting the variable alone on one side of the equal sign and a constant on the other side of the equation.

$$2x-6=14 \qquad x+3=4x-15$$
$$2x=8 \qquad 18=3x$$
$$x=4 \qquad 6=x$$

Second Degree Equations

Second Degree Equations contain a polynomial with a second degree variable term as its **highest power.** They are also called Quadratic Equations.

$x^2+2x-6=0$ $\qquad$ $x^2+2x=10$ $\qquad$ $x^2-2=7$

Second Degree Equations in Standard Form

The **Standard Form** for these equations is to write the second degree x^2 term first, the first degree term x second (if there is one) and the constant term last (if there is one) with the polynomial **set equal to zero.** It is also standard to have the second degree term x^2 have a positive coefficient.

$x^2-3x-10=0$ $\qquad$ $3x^2+6x+8=0$ $\qquad$ $x^2-36=0$ (no x term) $\qquad$ $3x^2-9x=0$ (no constant term)

Second Degree Equations have 2 solutions

Second Degree Equations have 2 values for x that will make the equation true if they are substituted into the equation in place of x.

Example 1

$x=-2$ or $x=5$

are both solutions to

$x^2-3x-10=0$

Check: $x=-2$

$$(-2)^2-3(-2)-10=0$$
$$4+6-10=0$$
$$0=0$$

Check: $x=5$

$$(5)^2-3(5)-10=0$$
$$25-15-10$$
$$0=0$$

Example 2

$x=6$ or $x=-6$

are both solutions to

$x^2-36=0$

Check: $x=6$

$$(6)^2-36=0$$
$$36-36=0$$
$$0=0$$

Check: $x=-6$

$$(-6)^2-36=0$$
$$36-36-10$$
$$0=0$$

Solving Second Degree Equations

First Degree Equations were solved by getting the variable alone on one side of the equal sign and a constant on the other side of the equation. This technique will not work with second degree equations. These equations have both an x^2 term and an x term and getting either term on one side of the equation means the other variable term will be on the other side with the constant. We must find a new way to solve these Second Degree Equations.

There are many different types of numbers that are solutions to Second Degree Equations

The solutions for First Degree Equations are always **rational numbers.** This means the solutions are values of x that are integers or positive and negative fractions like -2 or 3 or $\frac{3}{4}$ or $\frac{-5}{2}$. This is not true for second degree equations. The solutions for second degree equations **can be rational numbers** but they can also be **irrational numbers** like $\sqrt{6}$, $\sqrt{7}$, $2\sqrt{5}$ or even **imaginary numbers** like $\sqrt{-3}$.

There are several different techniques to solve second degree equations. If the solution is a **rational number** then solutions can be found using a factoring method. Other methods are used in the solution is a he **Zero Factor Rule like** $\sqrt{-3}$.

We will limit our Second Degree Equations to ones that contain second degree polynomials that can be factored.

The second degree equations we will solve in this chapter are limited to second degree polynomials **that can be factored.** A special rule called the **Zero Factor Rule** is used that requires polynomials **that can be factored.**

The solutions to equations involving the **Zero Factor Rule** will be **Rational Numbers** like -2 or 3 or $\frac{3}{4}$ or $\frac{-5}{2}$. Methods for finding solutions to second degree equations that cannot be factored and have solutions that are **irrational number** or **imaginary numbers** will be developed in a later chapter.

Zero Factor Rule

This rule helps us solve **equations** that have **products (factors)** that are **equal to zero.**

$(x-3)(x+4)=0 \qquad (x+2)(x-2)=0 \qquad (4x)(x-6)=0 \qquad x(5x+2)=0$

The zero product rule says

if you have an equation with two first degree terms whose **product (factors) equal zero** then
either the first factor is equal to zero
or the second factor is equal to zero

If the polynomial is a second degree equation then the polynomial will have exactly **two factors.** Each factor will be a first degree term. If the **product of the two factors equals zero** then you can use the Zero Product Rule to find the **two numbers** that are solutions to the second degree equation.

Solving 2nd Degree Equations using the Zero Factor Rule

Step 1: Be sure that the problem has an equation with **two factors whose product is zero.**

Step 2: Set each factor with an x term equal to zero. This will give you two separate equations.

Step 3: Solve each equation separately. Each solution is a solution to the original equation.

Step 4: You can check the 2 solutions by substituting either of the numbers into the original equation and checking to see that they make original make the equation true.

Solve each equation.

Example 1

$(x-5)(x+6)=0$

then

$x-5=0$ or $x+6=0$

and solving each equation for x gives

$x=5$ or $x=-6$

Check $x=5$: $(5-5)(5+6)=0$, $(0)(11)=0$

Check $x=-6$: $(-6-5)(-6+6)=0$, $(-11)(0)=0$

Both $x=5$ and $x=6$ are solutions to $(x-5)(x+6)=0$

Example 2

$(4x)(x-2)=0$

then

$4x=0$ or $x-2=0$

and solving each equation for x gives

$x=0$ or $x=2$

Check $x=0$: $(4\bullet 0)(0-2)=0$, $(0)(-2)=0$

Check $x=2$: $(4\bullet 2)(2-2)=0$, $(8)(0)=0$

Both $x=0$ and $x=2$ are solutions to $(4x)(x-2)=0$

Standard Form of a Second Degree Equation

The standard form of a second degree equation requires the x^2 term to be written first and be positive. The x term is written second and the constant term is written last. That expression is set equal to 0.

Standard Form Examples

$x^2 + 6x - 7 = 0$ $\qquad$ $3x^2 + 6x = 0$ $\qquad$ $25x^2 - 4 = 0$

Solving a Second Degree Equation

If we factor a quadratic equation **in standard form** we get two factors that have a product of zero.

$x^2 + 6x - 7 = 0$	$3x^2 + 6x = 0$	$25x^2 - 4 = 0$
$(x+7)(x-1) = 0$	$(3x)(x+2) = 0$	$(5x-2)(5x+2) = 0$

If we use the Zero Factor rule we can solve each of these equations.

$(x+7)(x-1) = 0$	$(3x)(x+6) = 0$	$(x-3)(x+3) = 0$
$x+7=0$ or $x-1=0$	$3x=0$ or $x+6=0$	$x-3=0$ or $x+3=0$
$x=-7$ or $x=1$	$x=0$ or $x=-6$	$x=3$ or $x=-3$

Both numbers are solutions to the original second degree equation.

To Solve a Second Degree Equation for x:

Step 1: Get the terms in **Standard Form** and **set equal to zero.**

Step 2. **Factor (**Factor out the **GCF**, The **Difference of 2 Perfect Squares**, **Easy Trinomials**)

Step 3. Set each factor **that has an x term** equal to zero.

Step 4. **Solve** each equation **for x.** Remember that second degree equations have 2 solutions.

Example 1

Solve $6x^2 - 12x = 0$

factor

$6x(x-2) = 0$

Set each factor = to 0

$6x = 0 \qquad x - 2 = 0$

Solve each equation for x

$x = 0$ or $x = 2$

Example 2

Solve $10x^2 + 5x = 0$

factor

$5x(2x+1) = 0$

Set each factor = to 0

$5x = 0 \qquad 2x + 1 = 0$

Solve each equation for x

$x = 0$ or $x = \frac{-1}{2}$

Both numbers are solutions to the original second degree equation.

Example 3

Solve $4x^2 - 25 = 0$

factor

$(2x-5)\,(2x+5) = 0$

Set each factor = to 0

$2x - 5 = 0 \qquad 2x + 5 = 0$

Solve each equation for x

$x = \frac{5}{2}$ or $x = \frac{-5}{2}$

Example 4

Solve $x^2 + 8x + 15 = 0$

factor

$(x+5)\,(x+3) = 0$

Set each factor = to 0

$x + 5 = 0 \qquad x + 3 = 0$

Solve each equation for x

$x = -5$ or $x = -3$

Both numbers are solutions to the original second degree equation.

Example 5

Solve $8x^2 - 32 = 0$

factor

$8(x^2 - 4) = 0$

$8(x-2)\,(x+2) = 0$

Set each factor with an x term = to 0

$x - 2 = 0 \qquad x + 2 = 0$

Solve each equation for x

$x = 2$ or $x = -2$

Example 6

Solve $2x^2 - 18 = 0$

factor

$2(x^2 - 9) = 0$

$2(x-3)\,(x+3) = 0$

Set each factor with an x term = to 0

$x - 3 = 0 \qquad x + 3 = 0$

Solve each equation for x

$x = 3$ or $x = -3$

Both numbers are solutions to the original second degree equation.

Example 7

Solve $x^2 - 6x - 7 = 0$

factor

$(x-7)(x+1) = 0$

Set each factor = to 0

$x - 7 = 0 \qquad x + 1 = 0$

Solve each equation for x

$x = 7$ or $x = -1$

Example 8

Solve $3x^2 - 2x - 4 = 0$

factor

$(x-2)(3x+4) = 0$

Set each factor = to 0

$x - 2 = 0 \qquad 3x + 4 = 0$

Solve each equation for x

$x = 2$ or $x = \frac{-4}{3}$

Example 9

Solve $8x^2 = 4x$

Put in standard form

$8x^2 - 4x = 0$

factor $8x^2 - 4x = 0$

$4x(2x-1) = 0$

Set each factor = to 0

$4x = 0 \qquad 2x - 1 = 0$

Solve each equation for x

$x = 0$ or $x = 1/2$

Example 10

Solve $16x^2 = 81$

Put in standard form

$16x^2 - 81 = 0$

factor $16x^2 - 81 = 0$

$(4x-9)(4x+9) = 0$

Set each factor = to 0

$4x - 9 = 0 \qquad 4x + 9 = 0$

Solve each equation for x

$x = \frac{9}{4}$ or $x = \frac{-9}{4}$

Example 11

Solve $x^2 = -9x - 14$

Put in standard form

$x^2 + 9x + 14 = 0$

factor $x^2 + 9x + 14 = 0$

$(x+7)(x+2) = 0$

Set each factor = to 0

$x + 7 = 0 \qquad x + 2 = 0$

Solve each equation for x

$x = -7$ or $x = -2$

Example 12

Solve $8x^2 = 4x$

Put in standard form

$8x^2 - 4x = 0$

factor $8x^2 - 4x = 0$

$4x(2x-1) = 0$

Set each factor = to 0

$4x = 0 \qquad 2x - 1 = 0$

Solve each equation for x

$x = 0$ or $x = 1/2$

Example 13

Solve $4x^2 = 25$

Put in standard form

$4x^2 - 25 = 0$

factor $4x^2 - 25 = 0$

$(2x-5)(2x+5) = 0$

Set each factor = to 0

$2x - 5 = 0 \qquad 2x + 5 = 0$

Solve each equation for x

$x = 5/2$ or $x = -5/2$

Example 14

Solve $x^2 = -9x - 14$

Put in standard form

$x^2 + 9x + 14 = 0$

factor $x^2 + 9x + 14 = 0$

$(x+7)(x+2) = 0$

Set each factor = to 0

$x + 7 = 0 \qquad x + 2 = 0$

Solve each equation for x

$x = -7$ or $x = -2$