

Section 7 – 4C: Factoring Trinomials of the form $Ax^2 \pm Bx \pm C$ where $A > 1$ using **The BOX Method**

Example 1: A trinomial with a positive last term.

Factor: $3x^2 + 7x + 2$

Step 1: Put the **first term** in the **upper left box** and the **last term** in the **lower right box**.

$3x^2$ the first term	the other middle term
one of the middle terms	2 the last term

Step 2: Find AC. Multiply the first and last terms, the $3x^2$ and the $+ 2$, to get $+ 6x^2$

Step 3: Find the 2 middle terms. The **last term** is **POSITIVE** so we want to find 2 middle terms that **multiply to $+ 6x^2$** and **ADD to $+ 7x$**

$$6 = \underline{1 \cdot 6} \quad 2 \cdot 3$$

The middle terms are $+ 1x$ and $+ 6x$

Put the $+ 1x$ and the $+ 6x$ into the two boxes for the **middle terms**. Include their signs.

Step 4. List the GCF of each row and column. Note how the signs for the GCF of the **bottom row** and **right column** are determined.

List the GCF of the two terms in the **top row** to the left of that row

$3x$	$3x^2$	$+ 6x$
$+ 1$	$+ 1x$	$+ 2$

List the GCF of the two terms in the **bottom row** to the left of that row.

The **sign for the GCF** of the **bottom row** is the sign of the term in the **bottom left box**.

The **sign for the GCF** of the **right column** is the sign of the term in the **upper right box**.

List the GCF of the two terms in the **left column** and put it above that column.

$3x$	$+ 2$
$6x^2$	$+ 6x$
$+ 1x$	$+ 2$

List the GCF of the two terms in the **right column** and put it above that column.

The completed box is shown to the right

	x	$+ 2$
$3x$	$3x^2$	$+ 6x$
$+ 1$	$+ 1x$	$+ 2$

GCF

$3x^2 + 7x + 2$
factors into
 $(3x + 1)(x + 2)$

Example 2: A trinomial with a positive last term.

Factor: $6x^2 - 11x + 3$

Step 1: Put the **first term** in the **upper left box** and the **last term** in the **lower right box**.

$6x^2$ the first term	the other middle term
one of the middle terms	3 the last term

Step 2: Find AC. Multiply the first and last terms, the $6x^2$ and the $+3$, to get $+18x^2$

Step 3: Find the 2 middle terms. The **last term** is **POSITIVE** so we want to find 2 middle terms that **multiply to $+18x^2$** and **ADD to $-11x$**

$$18 = 1 \cdot 18 \quad \underline{2 \cdot 9} \quad 3 \cdot 6$$

The middle terms are $-2x$ and $-9x$

Put the $-2x$ and $-9x$ into the two boxes for the **middle terms**. Include their signs.

Step 4. List the GCF of each row and column. Note how the signs for the GCF of the bottom row and left column are determined.

List the GCF of the two terms in the **top row** to the left of that row

$2x$	$6x^2$	$-2x$
-3	$-9x$	$+3$

List the GCF of the two terms in the **bottom row** to the left of that row.

The **sign for the GCF** of the **bottom row** is the sign of the term in the **bottom left box**.

The **sign for the GCF** of the **right column** is the sign of the term in the **upper right box**.

List the GCF of the two terms in the **left column** and put it above that column.

$3x$	-1
$6x^2$	$-2x$
$-9x$	$+3$

List the GCF of the two terms in the **right column** and put it above that column.

The completed box is shown to the right

	$3x$	-1
$2x$	$6x^2$	$-2x$
-3	$-9x$	$+3$

← GCF

$6x^2 - 11x + 3$
factors into
 $(2x - 3)(3x - 1)$

Example 3: A trinomial with a negative last term.

Factor: $8x^2 - 10x - 3$

Step 1: Put the **first term**, the $8x^2$, in the **upper left square** and the **last term**, the -3 , in the **lower right square**.

$8x^2$ the first term	the other middle term
one of the middle terms	-3 the last term

Step 2: Find AC. Multiply the two outer terms, the $8x^2$ and the -3 , to get $-24x^2$

Step 3: Find the 2 middle terms. The **last term is NEGATIVE** so we need to find 2 middle terms that **multiply to $-24x^2$** and **SUBTRACT to $-10x$**

$$24 = 1 \cdot 24 \quad \underline{2 \cdot 12} \quad 3 \cdot 8 \quad 4 \cdot 6$$

The middle terms are $+2x$ and $-12x$

Put the $+2x$ and the $-12x$ into the two empty squares. Include the signs.

Step 4. Factor out the GCF and the correct sign to find the factors.

List the GCF of the two terms in the **top row** to the left of that row

$2x$	$6x^2$	$-2x$
-3	$-9x$	$+3$

List the GCF of the two terms in the **bottom row** to the left of that row.

The **sign for the GCF** of the **bottom row** is the sign of the term in the **bottom left box**.

The **sign for the GCF** of the **right column** is the sign of the term in the **upper right box**.

List the GCF of the two terms in the **left column** and put it above that column.

$3x$	-1
$6x^2$	$-2x$
$-9x$	$+3$

List the GCF of the two terms in the **right column** and put it above that column.

The completed box is shown to the right

	$4x$	-1
$2x$	$8x^2$	$+2x$
-3	$-12x$	-3

← GCF

$8x^2 - 10x - 3$
factors into
 $(2x - 3)(3x - 1)$

Example 4: A trinomial with a negative last term.

Factor: $18x^2 - 9x - 2$

Step 1: Put the **first term**, the $18x^2$, in the **upper left square** and the **last term**, the -2 , in the **lower right square**.

$18x^2$ the first term	the other middle term
one of the middle terms	-2 the last term

Step 2: Find AC. Multiply the two outer terms, the $18x^2$ and the -2 , to get $-36x^2$

Step 3: Find the 2 middle terms. The **Last Term is NEGATIVE** so we need to find 2 middle terms that **multiply to $-36x^2$** and **SUBTRACT to $-9x$**

$$36 = 1 \cdot 36 \quad 2 \cdot 18 \quad \underline{3 \cdot 12} \quad 4 \cdot 9 \quad 6 \cdot 6$$

The middle terms are $+3x$ and $-12x$

Put the $+3x$ and the $-12x$ into the two empty squares. Include the signs.

Step 4. Factor out the GCF and the correct sign to find the factors.

List the GCF of the two terms in the **top row** to the left of that row

$6x$	$18x^2$	$-12x$
$+1$	$+3x$	-1

List the GCF of the two terms in the **bottom row** to the left of that row.

The **sign for the GCF** of the **bottom row** is the sign of the term in the **bottom left box**.

The **sign for the GCF** of the **right column** is the sign of the term in the **upper right box**.

List the GCF of the two terms in the **left column** and put it above that column.

$3x$	-2
$18x^2$	$-12x$
$+3x$	-2

List the GCF of the two terms in the **right column** and put it above that column.

The completed box is shown to the right

	$3x$	-2
$6x$	$18x^2$	$-12x$
$+1$	$+3x$	-2

GCF

$18x^2 - 9x - 2$
factors into
 $(6x + 1)(3x - 2)$

Worked Examples with less explanation

Factor: $9x^2 - 9x + 2$

Step 1:

$9x^2$ the first term	the other middle term
one of the middle terms	$+ 2$ the last term

Step 2: Find AC. Multiply the two outer terms, the $9x^2$ and the $+ 2$, to get $+ 18x^2$

Step 3: Find the 2 middle terms. The **last term** is **POSITIVE** so we want to find 2 middle terms that **multiply to $+ 18x^2$** and **ADD to $- 9x$**

$$18 = 1 \cdot 18 \quad 2 \cdot 9 \quad \underline{3 \cdot 6}$$

The middle terms are $- 3x$ and $- 6x$

Step 4. Fill in the box. List each GCF with the correct sign. Write out the factors.

	$3x$	$- 2$	
$3x$	$9x^2$	$- 6x$	$9x^2 - 9x + 2$ factors into $(3x - 1) (3x - 2)$
$- 1$	$- 3x$	$+ 2$	
	← GCF		

Factor: $10x^2 - x - 2$

Step 1:

$10x^2$ the first term	the other middle term
one of the middle terms	$- 2$ the last term

Step 2: Find AC. Multiply the two outer terms , the $10x^2$ and the $- 2$, to get $- 20x^2$

Step 3: Find the 2 middle terms. The **last term** is **NEGATIVE** so we want to find 2 middle terms that **multiply to $- 20x^2$** and **SUBTRACT to $- x$**

$$20 = 1 \cdot 20 \quad 2 \cdot 10 \quad \underline{4 \cdot 5}$$

The middle terms are $+ 4x$ and $- 5x$

Step 4. Fill in the box. List each GCF with the correct sign. Write out the factors.

	$2x$	$- 1$	
$5x$	$10x^2$	$- 5x$	$10x^2 - x - 2$ factors into $(5x + 2) (2x - 1)$
$+ 2$	$+ 4x$	$- 2$	
	← GCF	↑	