

## Section 7 – 4B:

## Factoring Trinomials of the Form

$$Ax^2 \pm Bx \pm C \text{ where } A > 1$$

by

**The AC and Factor By Grouping Method**

$$\text{Easy Trinomials: } 1x^2 \pm Bx \pm C$$

The last section covered the topic of factoring second degree trinomials that had  $1x^2$  as the first term. These trinomials looked like  $1x^2 \pm bx \pm c$ . The factoring technique used in that section did not require multiple steps and many students develop their skills to the level where they can factor these trinomials in their head. For this reason we call these type of trinomials "**Easy Trinomials**" as in "easy to factor."

### Hard Trinomials:

Hard Trinomials have a first term of  $Ax^2$  where  $A > 1$ .

$$Ax^2 \pm Bx \pm C \text{ where } A > 1$$

### Examples of Hard Trinomials

$$6x^2 - 25x + 4$$

$$5x^2 - 16x + 15$$

$$12x^2 + x - 6$$

$$15x^2 - x - 6$$

Students are taught many different techniques to factor Hard Trinomials: Factoring by Grouping, The AC method, trying all the possible factors with FOIL, The Box Method, and many others. A search of the internet and u-tube will reveal many methods. Each of these techniques require 3 or 4 written steps. The amount of work required to factor these trinomials is why we call them "Hard Trinomials"

The best technique is the one you know and are good at. If you already know a method to factor these polynomials and are good at it then **continue to use that technique**. If you do not know a technique then the one that follows **may be considered**. It is as easy as any of the other choices, and seems to be one the easiest methods for many students to learn.

It seems every technique for factoring Hard Trinomials has a name. This method will be called

Factoring Hard Trinomials

by

**The AC and Factor By Grouping Method**

## The AC and Factor By Grouping Method

to factor

$$A x^2 \pm Bx + C \text{ where } C \text{ is positive}$$

### Example 1

$$\text{Factor: } 3x^2 + 7x + 2$$

Step 1: Multiply the two outer terms

$$3x^2 \text{ and the } 2 \text{ to get } +6x^2$$

$$\begin{array}{c} \overbrace{\hspace{1.5cm}} \\ 3x^2 + 7x + 2 \end{array}$$

Step 2: Find 2 terms that **multiply to  $+6x^2$**  and **add to  $+7x$**   
 **$+6x$  and  $+1x$**

Step 3: Replace the  $-9x$  in

$$2x^2 - 9x + 10 \text{ with}$$

$$\begin{array}{c} -5x - 4x \\ \downarrow \quad \downarrow \\ 2x^2 \quad \underline{\quad} \quad \underline{\quad} + 10 \end{array}$$

to get

$$2x^2 - 5x - 4x + 10$$

Step 4:  $3x^2 + 6x + 1x + 2$

Factor a GCF of  $3x$  out of the first two terms and  
Factor a GCF of  $1$  out of the last two terms

$$3x(x + 2) + 1(x + 2)$$

Factor a GCF of  $(x + 2)$  out of the two terms

$$(x + 2)(3x + 1)$$

$$\text{Factor: } 3x^2 + 7x + 2$$

$$(x + 2)(3x + 1)$$

## The AC – Factor By Grouping Method

### Example 2

Factor:  $2x^2 - 9x + 10$

Step 1: Multiply the two outer terms

$2x^2$  and the  $10$  to get  $+20x^2$

$$2x^2 - 9x + 10$$

Step 2: Find 2 terms that **multiply to  $+20x^2$**  and **add to  $-9x$**   
 **$-5x$  and  $-4x$**

Step 3: Replace the  $-9x$  in

$2x^2 - 9x + 10$  with

$-5x - 4x$

$$2x^2 \quad \underline{\quad} \quad \underline{\quad} \quad + 10$$

to get

$2x^2 - 5x - 4x + 10$

Step 4:  $2x^2 - 5x - 4x + 10$

Factor a GCF of  $x$  out of the first two terms and

Factor a GCF of  $-2$  out of the last two terms

$x(2x - 5) - 2(2x - 5)$

Factor a GCF of  $(2x - 5)$  out of the two terms

$(2x - 5)(x - 2)$

Factor:  $2x^2 - 9x + 10$

$(2x - 5)(x - 2)$

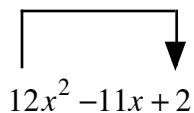
## The AC – Factor By Grouping Method

### Example 3

Factor:  $12x^2 - 11x + 2$

Step 1: Multiply the two outer terms

$12x^2$  and the  $2$  to get  $+24x^2$

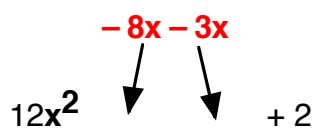

$$12x^2 - 11x + 2$$

Step 2: Find 2 terms that **multiply to  $+24x^2$**  and **add to  $-11x$**   
 **$-8x$  and  $-3x$**

Step 3: Replace the  $-11x$  in

$12x^2 - 11x + 2$  with

$-8x - 3x$


$$12x^2 \quad \underline{\quad} \quad \underline{\quad} \quad + 2$$

to get

$12x^2 - 8x - 3x + 2$

Step 4:  $12x^2 - 8x - 3x + 2$

Factor a GCF of  $4x$  out of the first two terms and  
Factor a GCF of  $-1$  out of the last two terms

$4x(3x - 2) - 1(3x - 2)$

Factor a GCF of  $(3x - 2)$  out of the two terms

$(3x - 2)(4x - 1)$

Factor:  $12x^2 - 11x + 2$

$(3x - 2)(4x - 1)$

## The AC – Factor By Grouping Method

to factor

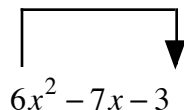
$$Ax^2 \pm Bx - C \quad \text{where } C \text{ is Negative}$$

### Example 4

$$\text{Factor: } 6x^2 - 7x - 3$$

Step 1: Multiply the two outer terms

$$6x^2 \text{ and the } -3 \text{ to get } -18x^2$$

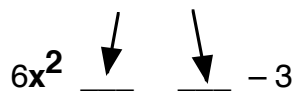

$$6x^2 - 7x - 3$$

Step 2: Find 2 terms that **multiply to  $-18x^2$**  and **subtract to  $-7x$**   
 **$-9x$  and  $+2x$**

Step 3: Replace the  **$-7x$**  in

$$6x^2 - 7x - 3 \quad \text{with}$$

$$-9x + 2x$$


$$6x^2 \quad \underline{\quad} \quad \underline{\quad} \quad -3$$

to get

$$6x^2 - 9x + 2x - 3$$

Step 4:  $6x^2 - 9x + 2x - 3$

**Factor a GCF of  $3x$  out of the first two terms and  
Factor a GCF of  $+1$  out of the last two terms**

$$3x(2x - 3) + 1(2x - 3)$$

**Factor a GCF of  $(2x - 3)$  out of the two terms**

$$(2x - 3)(3x + 1)$$

$$\text{Factor: } 6x^2 - 7x - 3$$

$$(2x - 3)(3x + 1)$$

## The AC – Factor By Grouping Method

to factor

$$Ax^2 \pm Bx - C \quad \text{where } C \text{ is Negative}$$

### Example 5

$$\text{Factor: } 12x^2 + 5x - 2$$

Step 1: Multiply the two outer terms

$$12x^2 \text{ and the } -2 \text{ to get } -24x^2$$

Step 2: Find 2 terms that **multiply to**  $-24x^2$  and **subtract to**  $+5x$   
 $+8x$  and  $-3x$

Replace the  $+5x$  in

$$12x^2 + 5x - 2 \quad \text{with}$$

$$\begin{array}{ccc} & +8x - 3x & \\ & \downarrow \quad \downarrow & \\ 12x^2 & \underline{\quad} \quad \underline{\quad} & -2 \end{array}$$

to get

$$12x^2 + 8x - 3x - 2$$

Step 4:

$$12x^2 + 8x - 3x - 2$$

Factor a GCF of  $4x$  out of the first two terms and  
Factor a GCF of  $-1$  out of the last two terms

$$4x(3x + 2) - 1(3x + 2)$$

Factor a GCF of  $(3x + 2)$  out of the two terms

$$(3x + 2)(4x - 1)$$

$$\text{Factor: } 12x^2 + 5x - 2$$

$$(3x + 2)(4x - 1)$$

## The AC – Factor By Grouping Method

to factor

$$Ax^2 \pm Bx - C \quad \text{where } C \text{ is Negative}$$

### Example 6

$$\text{Factor: } 8x^2 - 10x - 3$$

Step 1: Multiply the two outer terms

$$8x^2 \text{ and the } -3 \text{ to get } -24x^2$$

Step 2: Find 2 terms that **multiply to**  $-24x^2$  and **subtract to**  $-10x$   
 $-12x$  and  $+2x$

Step 3: Replace the  $-10x$  in

$$8x^2 - 10x - 3 \quad \text{with}$$

$$-12x + 2x$$

$$8x^2 \quad \downarrow \quad \downarrow \quad -3$$

to get

$$8x^2 - 12x + 2x - 3$$

Step 4:  $8x^2 - 12x + 2x - 3$

**Factor a GCF of  $4x$  out of the first two terms and**

**Factor a GCF of  $+1$  out of the last two terms**

$$4x(2x - 3) + 1(2x - 3)$$

**Factor a GCF of  $(2x - 3)$  out of the two terms**

$$(2x - 3)(4x + 1)$$

$$\text{Factor: } 8x^2 - 10x - 3$$

$$(2x - 3)(4x + 1)$$

## Some Trinomials DO NOT Factor (DNF)

Many **trinomials** cannot be factored.

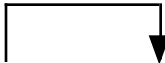
We write **DOES NOT Factor or Prime or DNF** if that is the case.

### Example 7

Factor:  $3x^2 + 2x + 4$

Step 1: Multiply the two outer terms

$3x^2$  and the  $+4$  to get  $+12x^2$



$3x^2 + 2x + 4$

Step 2: Find 2 terms that **multiply to  $+12x^2$**  and **add to  $+2x$**   
**there are no numbers that work**  
**so the hard trinomial Does Not Factor DNF**

Factor:  $3x^2 + 2x + 4$

Does not Factor

DNF

Prime

**The GCF must be taken out first** before factoring the hard trinomial

To factor any polynomial completely you should **factor the GCF out FIRST** if the polynomial has a GCF. If the polynomial remaining inside the parentheses is a trinomial that can be factored then the complete factored answer is the product of the GCF and the factored trinomial.

### Example 8

Factor:  $6x^2 - 26x + 8$

Factor out the GCF of 2

$$2(3x^2 - 13x + 4)$$

now factor the hard trinomial

$$3x^2 - 13x + 4$$

Step 1: Multiply the two outer terms

$$3x^2 \text{ and the } + 4 \text{ to get } + 12x^2$$

$$3x^2 - 13x + 4$$

Step 2: Find 2 terms that **multiply to  $+ 12x^2$**  and **add to  $- 13x$**   
 $- 12x$  and  $- 1x$

Step 3: Replace the  $- 13x$  in

$$3x^2 - 13x + 4 \text{ with}$$

$$- 12x - 1x$$

$$3x^2 \quad \underline{\quad} \quad \underline{\quad} + 4$$

to get

$$3x^2 - 12x - 1x + 4$$

Step 4:

$$3x^2 - 12x - 1x + 4$$

**Factor a GCF of  $3x$  out of the first two terms and**

**Factor a GCF of  $- 1$  out of the last two terms**

$$3x(x - 4) - 1(x - 4)$$

**Factor a GCF of  $(x - 4)$  out of the two terms**

$$(x - 4)(3x - 1)$$

we factored out a 2 as the GCF of  $6x^2 - 26x + 8$  to get  $2(3x^2 - 13x + 4)$

so the final factored answer is

$$2(x - 4)(3x - 1)$$