

Section 7 – 4A: Factoring Trinomials of the Form

$$Ax^2 \pm Bx \pm C \quad \text{where } A > 1$$

$$\text{Easy Trinomials: } 1x^2 \pm bx \pm c$$

The last section covered the topic of factoring second degree trinomials that had $1x^2$ as the first term. These trinomials looked like $1x^2 \pm bx \pm c$. The factoring technique used in that section did not require multiple steps and many students develop their skills to the level where they can factor these trinomials in their head. For this reason we call these type of trinomials "**Easy Trinomials**" as in "easy to factor."

Hard Trinomials:

Hard Trinomials have a first term of Ax^2 where $A > 1$.

$$Ax^2 \pm Bx \pm C \quad \text{where } A > 1$$

Examples of Hard Trinomials

$$6x^2 - 25x + 4$$

$$5x^2 - 16x + 15$$

$$12x^2 + x - 6$$

$$15x^2 - x - 6$$

Students are taught many different techniques to factor Hard Trinomials: Factoring by Grouping, The AC method, trying all the possible factors with FOIL, The Box Method, and many others. A search of the internet and u-tube will reveal many methods. Each of these techniques require 3 or 4 written steps. The amount of work required to factor these trinomials is why we call them "Hard Trinomials"

The best technique is the one you know and are good at. If you already know a method to factor these polynomials and are good at it then **continue to use that technique**. If you do not know a technique then the one that follows **may be considered**. It is as easy as any of the other choices, and seems to be one the easiest methods for many students to learn.

It seems every technique for factoring Hard Trinomials has a name. This method will be called

Factoring Hard Trinomials by Creating an Easy Trinomial Method

Note: This method requires that **the GCF must be taken out first before factoring the hard trinomial**.

An internet search may using the term "Factoring trinomials by the Glide and Slide Method" will have several links to this topic.

Creating an Easy Trinomial Method

Example 1

$$\text{Factor: } 3x^2 + 7x + 2$$

Step 1: The **GCF must be taken out first (if there is one)** before factoring the hard trinomial.

Step 2: Create an Easy Trinomial by moving the coefficient of the $3x^2$ term to the end of the trinomial and multiplying the **3** and the 2

$$\begin{array}{c} \boxed{} \\ 3x^2 + 7x + 2 \cdot \mathbf{3} \end{array}$$

to get the easy trinomial

$$x^2 + 7x + 6$$

Step 3: Factor the easy trinomial by finding the 2 numbers that **multiply to + 6 and add to + 7**

$$+ 6 \text{ and } + 1$$

$$(x + 6)(x + 1)$$

Step 4: In Step 1 you multiplied the constant 2 by the **3** that was the coefficient of the $3x^2$ term. Now **divide BOTH of the constants** in $(x + 6)(x + 1)$ by **3**

$$\left(x + \frac{6}{3}\right)\left(x + \frac{1}{3}\right)$$

reduce each fraction

$$(x + 2)\left(x + \frac{1}{3}\right)$$

Step 5: "glide" the denominator of each fraction (if there is one) to the front of the x term

$$(x + 2)\left(\mathbf{3}x + \frac{1}{3}\right)$$

$$(x + 2)(3x + 1)$$

$$\text{Factor: } 3x^2 + 7x + 2$$

$$(x + 2)(3x + 1)$$

Creating an Easy Trinomial Method

Example 3

$$\text{Factor: } 6x^2 - x - 1$$

Step 1: The **GCF must be taken out first (if there is one)** before factoring the hard trinomial.

Step 2: Create an Easy Trinomial by moving the coefficient of the $6x^2$ term to the end of the trinomial and multiplying the **6** and the -1

$$\begin{array}{c} \boxed{} \quad \downarrow \\ 6x^2 - x - 1 \cdot \mathbf{6} \\ \text{to get the easy trinomial} \end{array}$$

$$x^2 - x - 6$$

Step 3: Factor the easy trinomial by finding the 2 numbers that **multiply to -6 and subtract to -1**

$$-3 \text{ and } +2$$

$$(x - 3)(x + 2)$$

Step 4: In step 1 you multiplied the constant -1 by the **6** that was in front of the x^2 term. Now **divide BOTH of the constants** in $(x - 3)(x + 2)$ by **6**

$$\left(x - \frac{3}{6}\right)\left(x + \frac{2}{6}\right)$$

reduce each fraction

$$\left(x - \frac{1}{2}\right)\left(x + \frac{1}{3}\right)$$

Step 5: "glide" the denominator of each fraction (if there is one) to the front of the x term

$$\begin{array}{c} \left(\mathbf{2}x - \frac{1}{2}\right)\left(\mathbf{3}x + \frac{1}{3}\right) \\ \uparrow \quad \downarrow \quad \uparrow \quad \downarrow \\ \left(2x - 1\right)\left(3x + 1\right) \end{array}$$

$$(2x - 1)(3x + 1)$$

$$\text{Factor: } 6x^2 - x - 6$$

$$(2x - 1)(3x + 1)$$

Creating an Easy Trinomial Method

Example 4

$$\text{Factor: } 16x^2 - 8x + 1$$

Step 1: The **GCF must be taken out first (if there is one)** before factoring the hard trinomial.

Step 2: Create an Easy Trinomial by moving the coefficient of the $16x^2$ term to the end of the trinomial and multiplying the **16** and the -1

$$\begin{array}{c} \uparrow \quad \quad \quad \downarrow \\ 16x^2 - 8x - 1 \bullet 16 \end{array}$$

to get the easy trinomial

$$x^2 - 8x - 16$$

Step 3: Factor the easy trinomial by finding the 2 numbers that **multiply to -16 and subtract to -8**

$$-4 \text{ and } -4$$

$$(x - 4)(x - 4)$$

Step 4: In step 1 you multiplied the constant -1 by the **16** that was in front of the x^2 term. Now **divide BOTH of the constants** in $(x - 4)(x - 4)$ by **16**

$$\left(x - \frac{4}{16}\right)\left(x - \frac{4}{16}\right)$$

reduce each fraction

$$\left(x - \frac{1}{4}\right)\left(x - \frac{1}{4}\right)$$

Step 5: "glide" the denominator of each fraction (if there is one) in front of the x term

$$\begin{array}{c} \uparrow \quad \quad \downarrow \quad \quad \uparrow \quad \quad \downarrow \\ \left(4x - \frac{1}{4}\right)\left(4x - \frac{1}{4}\right) \end{array}$$

$$(4x - 1)(4x - 1)$$

$$\text{Factor: } 16x^2 - 8x + 1$$

$$(4x - 1)(4x - 1)$$

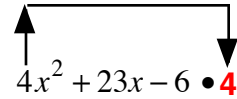
Creating an Easy Trinomial Method

Example 5

$$\text{Factor: } 4x^2 + 23x - 6$$

Step 1: The **GCF must be taken out first (if there is one)** before factoring the hard trinomial.

Step 2: Create an Easy Trinomial by moving the coefficient of the $4x^2$ term to the end of the trinomial and multiplying the **4** and the -6

$$4x^2 + 23x - 6 \bullet 4$$


to get the easy trinomial

$$x^2 + 23x - 24$$

Step 3: Factor the easy trinomial by finding the 2 numbers that **multiply to -24 and subtract to $+23$**

$$-24 \text{ and } +1$$

$$(x + 24)(x - 1)$$

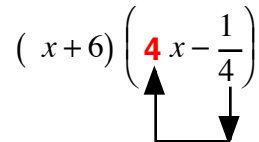
Step 4: In step 1 you multiplied the constant -6 by the **4** that was in front of the x^2 term. Now **divide BOTH of the constants** in $(x + 24)(x - 1)$ by **4**

$$\left(x + \frac{24}{4}\right)\left(x - \frac{1}{4}\right)$$

reduce each fraction

$$(x + 6)\left(x - \frac{1}{4}\right)$$

Step 5: "glide" the denominator of each fraction (if there is one) in front of the x term

$$(x + 6)\left(4x - \frac{1}{4}\right)$$


$$(x + 6)(4x - 1)$$

The GCF must be taken out first before factoring the hard trinomial

To factor any polynomial completely you should **factor the GCF out FIRST** if the polynomial has a GCF. If the polynomial remaining inside the parentheses is a trinomial that can be factored then the complete factored answer is the product of the GCF and the factored trinomial.

Example 6

Factor: $6x^2 + 26x + 8$

The **GCF must be taken out first (if there is one)** before factoring the hard trinomial.

Factor: $6x^2 + 26x + 8$

Factor out the GCF of 2

$$2(3x^2 + 13x + 4)$$

now factor the hard trinomial

$$3x^2 + 13x + 4$$

Factor: $3x^2 + 13x + 4$

Multiply the 3 by the 4

to get the easy trinomial

$$x^2 + 13x + 12$$

factor

$$(x + 12)(x + 1)$$

Divide both constants by 3

$$\left(x + \frac{12}{3}\right)\left(x + \frac{1}{3}\right)$$

reduce the $\frac{12}{3}$ in the first factor

and glide the 3 in front of the x in the second factor

$$(x + 4)(3x + 1)$$

be sure to include the GCF you factored out

in Step 1 in the final answer

Answer: $2(x + 4)(3x + 1)$

Some Trinomials DO NOT Factor (DNF)

Many **trinomials** cannot be factored.

We write **DOES NOT Factor** or **Prime** or **DNF** if that is the case.

Example 7

Factor: $3x^2 + 2x + 4$

Step 2: Create an Easy Trinomial by moving the coefficient of the $3x^2$ term to the end of the trinomial and multiplying the **3** and the $+4$

$$3x^2 + 2x + 4 \bullet 3$$

to get the easy trinomial

$$x^2 + 2x + 12$$

Step 3: Factor the easy trinomial by finding the 2 numbers that **multiply to 12 and add to +2**

there are **no numbers that work**
so the trinomial **DOES NOT FACTOR**

We can also write **DNF** or **Prime**