Section 7 – 3: Factoring Trinomials of the Form

$1x^2 \pm Bx \pm C$

The FOIL process changes a product of 2 binomials into a polynomial. The reverse process starts with a polynomial and finds the 2 binomials whose product will be that polynomial. The process of changing a polynomial into **a product of 2 binomials** is called **factoring a polynomial**.

This unit will look at factoring two types of trinomials.

A second degree **Trinomial** that has $1x^2$ as the first term and **ends with a positive number**
C is positive
$1x^2 \pm Bx + C$

A second degree **Trinomial** that has $1x^2$ as the first term and **ends with a negative number**
C is negative
$1x^2 \pm Bx - C$

The technique for factoring each type is similar. We will start with the type where **C is positive.**

How to factor $x^2 \pm Bx + C$ If the Trinomial **Ends in a Positive Number**

If the last term of $x^2 \pm Bx + C$ **is positive**

then we need two numbers **D** and **E** that
MULTIPLY to give + C
and ADD to give B

and the factors are written as

$(x \pm D)\ (x \pm E)$

Example 1

Factor $x^2 - 9x + 20$
into $(x \pm D)\ (x \pm E)$

$1 \cdot 20$
$2 \cdot 10$
$4 \cdot 5$

$x^2 - 9x + 20$

we need two numbers
D and **E** that
multiply to + 20
and **add to – 9**

– 4 and **– 5** work

Answer: $(x-4)\ (x-5)$

Example 2

Factor $x^2 + 9x + 18$
into $(x \pm D)\ (x \pm E)$

$1 \cdot 18$
$2 \cdot 9$
$3 \cdot 6$

$x^2 + 9x + 18$

we need two numbers
D and **E** that
multiply to + 18
and **add to + 9**

+ 3 and **+ 6** work

Answer: $(x+3)\ (x+6)$

Example 3

Factor $x^2 - 7x + 12$
into $(x \pm D)\ (x \pm E)$

$1 \cdot 12$
$2 \cdot 6$
$3 \cdot 4$

$x^2 - 7x + 12$

we need two numbers
D and **E** that
multiply to + 12
and **add to – 7**

– 3 and **– 4** work

Answer: $(x-3)\ (x-4)$

Example 4

Factor $x^2 + 12x + 20$

into (x ± D) (x ± E)

$1 \cdot 20$

$2 \cdot 10$

$4 \cdot 5$

$x^2 + 12x + 20$

we need two numbers D and E that **multiply to + 20** and **add to + 12**

+ 2 and **+ 10** work

Answer: $(x+2)\ (x+10)$

Example 5

Factor $x^2 - 19x + 18$

into (x ± D) (x ± E)

$1 \cdot 18$

$2 \cdot 9$

$3 \cdot 6$

$x^2 - 19x + 18$

we need two numbers D and E that **multiply to + 18** and **add to – 19**

– 1 and **– 18** work

Answer: $(x-1)\ (x-18)$

Example 6

Factor $x^2 + 8x + 12$

into (x ± D) (x ± E)

$1 \cdot 12$

$2 \cdot 6$

$3 \cdot 4$

$x^2 + 8x + 12$

we need two numbers D and E that **multiply to + 12** and **add to + 8**

+ 2 and **+ 6** work

Answer: $(x+2)\ (x+6)$

Note: The order of the factors does not matter. Either $(x+2)\ (x+10)$ or $(x+10)\ (x+2)$ is correct.

Note: If none of the products of the last term work then the trinomial does not factor. We write “Does Not Factor” or **DNF or prime.**

Example 7

Factor $x^2 - 21x + 20$

into (x ± D) (x ± E)

$1 \cdot 20$

$2 \cdot 10$

$4 \cdot 5$

$x^2 - 21x + 20$

we need two numbers D and E that **multiply to + 20** and **add to – 21**

– 1 and **– 20** work

$(x-1)\ (x-20)$

Answer: $(x-1)\ (x-20)$

Example 8

Factor $x^2 + 11x + 18$

into (x ± D) (x ± E)

$1 \cdot 18$

$2 \cdot 9$

$3 \cdot 6$

$x^2 + 11x + 18$

we need two numbers D and E that **multiply to + 18** and **add to + 11**

+ 2 and **+ 9** work

$(x+2)\ (x+9)$

Answer: $(x+2)\ (x+9)$

Example 9

Factor $x^2 - 13x + 12$

into (x ± D) (x ± E)

$1 \cdot 12$

$2 \cdot 6$

$3 \cdot 4$

$x^2 - 13x + 12$

we need two numbers D and E that **multiply to + 12** and **add to – 13**

– 1 and **– 12** work

$(x-1)\ (x-12)$

Answer: $(x-1)\ (x-12)$

Note: The order of the factors does not matter. Either $(x-1)\ (x-20)$ or $(x-20)(x-1)$ is correct.

Note: If none of the products of the last term work then the trinomial does not factor. We write “Does Not Factor” or **DNF or prime.**

How to factor $x^2 \pm Bx - C$ If the Trinomial **Ends in a Negative Number**

If the last term of $x^2 \pm Bx - C$ **is positive**

then we need two numbers **D** and **E** that
MULTIPLY to give + C
and SUBTRACT to give B

and the factors are written as

(x ± D) (x ± E)

Example 1

Factor $x^2 - x - 20$
into **(x ± D) (x ± E)**

$1 \cdot 20$
$2 \cdot 10$
$4 \cdot 5$

$x^2 - x - 20$

we need two numbers
D and **E** that
multiply to – 20
and **subtract to – 1**

– 5 and **4** work

Answer: $(x-5)\ (x+4)$

Example 2

Factor $x^2 + 3x - 18$
into **(x ± D) (x ± E)**

$1 \cdot 18$
$2 \cdot 9$
$3 \cdot 6$

$x^2 + 3x - 18$

we need two numbers
D and **E** that
multiply to – 18
and **subtract to + 3**

+ 6 and **– 3** work

Answer: $(x+6)\ (x-3)$

Example 3

Factor $x^2 - 4x - 12$
into **(x ± D) (x ± E**

$1 \cdot 12$
$2 \cdot 6$
$3 \cdot 4$

$x^2 - 4x - 12$

we need two numbers
D and **E** that
multiply to – 12
and **subtract to – 4**

– 6 and **+ 2** work

Answer: $(x-6)\ (x+2)$

Note: The order of the factors does not matter. Either $(x-1)\ (x-20)$ or $(x-20)(x-1)$ is correct.

Note: If none of the products of the last term work then the trinomial does not factor. We write "Does Not Factor" or **DNF or prime.**

Example 4

Factor $x^2 - 3x - 28$

into (x ± D) (x ± E)

$1 \cdot 28$

$2 \cdot 14$

$4 \cdot 7$

$x^2 + 12x - 28$

we need two numbers **D** and **E** that **multiply to – 28** and **subtract to – 3**

+ 4 and **– 7** work

Answer: $(x+4)\ (x-7)$

Example 5

Factor $x^2 - 15x - 16$

into (x ± D) (x ± E)

$1 \cdot 16$

$2 \cdot 8$

$4 \cdot 4$

$x^2 - 15x - 16$

we need two numbers **D** and **E** that **multiply to – 16** and **subtract to – 15**

– 16 and **+ 1** work

Answer: $(x-16)\ (x+1)$

Example 6

Factor $x^2 + 13x - 30$

into (x ± D) (x ± E)

$1 \cdot 30$

$5 \cdot 6$

$2 \cdot 15$

$x^2 + 13x - 30$

we need two numbers **D** and **E** that **multiply to – 30** and **subtract to + 13**

+ 15 and **– 2** work

Answer: $(x+15)\ (x-2)$

Example 7

Factor $x^2 - 3x - 40$

into (x ± D) (x ± E)

$1 \cdot 40$

$2 \cdot 20$

$4 \cdot 10$

$5 \cdot 8$

$x^2 - 3x - 40$

we need two numbers **D** and **E** that **multiply to –40** and **subtract to – 3**

– 8 and + 5 work

Answer: $(x-8)\ (x+5)$

Example 8

Factor $x^2 + 5x - 24$

into (x ± D) (x ± E)

$1 \cdot 24$

$2 \cdot 12$

$3 \cdot 8$

$4 \cdot 6$

$x^2 + 5x - 24$

we need two numbers **D** and **E** that **multiply to –24** and **subtract to + 5**

+ 8 and **– 3** work

Answer: $(x+8)\ (x-3)$

Example 9

Factor $x^2 - 8x - 48$

into (x ± D) (x ± E)

$1 \cdot 48$

$2 \cdot 24$

$3 \cdot 16$

$4 \cdot 12$

$6 \cdot 8$

$x^2 - 8x - 48$

we need two numbers **D** and **E** that **multiply to –48** and **subtract to – 8**

– 12 and **+ 4** work

Answer: $(x-12)\ (x+4)$

Note: The order of the factors does not matter. Either $(x-1)\ (x-20)$ or $(x-20)(x-1)$ is correct.

Note: If none of the products of the last term work then the trinomial does not factor. We write "Does Not Factor" or **DNF or prime.**

Factoring Completely: Putting It all Together

Many Polynomials have a GCF that can be factored out and the expression left inside the parenthesis can then also be factored. This requires **two steps to completely factor the initial polynomial.**

Step 1: The first step in any factoring process is to **factor out the GCF** if there is one:

Example 1

$27x^2 - 12$

$= 3(9x^2 - 4)$

Example 2

$2x^2 - 8x - 10$

$= 2(x^2 - 4x - 5)$

Example 3

$4x^2 + 16x + 12$

$= 4(x^2 + 4x + 3)$

At this point you will have either a **Binomial** or a **Trinomial inside the parenthesis.**

Step 2: Factor the expression inside the parenthesis if possible:

If there is a Binomial inside the parenthesis:

1. If it is the **Difference of Two Perfect Squares** then factor using $(a^2 - b^2) = (a+b)(a-b)$.

Example 4

$2(x^2 - 9)$

$= 2(x+3)(x-3)$

Example 5

$3x(4x^2 - 1)$

$= 3x(2x-1)(2x+1)$

2. If it is the **Sum of Two Perfect Squares** it cannot be factored. **Factoring is complete.**

$2(x^2 + 9)$ Factoring is complete

$3x^2(4x^2 + 25)$ Factoring is complete

If there is a Trinomial inside the parenthesis:

1. If the sign of the **last term of the trinomial is positive** look for the factors of the last term that **multiply to give the last term** and **add to give the middle term** of the trinomial.

2. If the sign of the **last term of the trinomial is negative** look for the factors of the last term that **multiply to give the last term** and **subtract to give the middle term** of the trinomial.

Example 6

$2x(x^2 - x + 12)$

$= 2x(x-4)(x+3)$

Example 7

$7x(x^2 + 3x - 10)$

$= 7x(x+5)(x-2)$

If the trinomial inside the parenthesis is not any of the types listed above we will not discuss how it may factor until a later course. For the purposes of this course **factoring is complete.**

Factoring a Polynomial Completely

To factor any polynomial completely you should **factor the GCF out FIRST** if the polynomial has a GCF. If the polynomial remaining inside the parentheses is a trinomial that can be factored then the complete factored answer is the product of the GCF and the factored trinomial.

1. Factor out the **GCF** if there is one
2. Factor the polynomial inside the parentheses If it can be factored.
3. The final answer is t**he product of the GCF and the other factors.**

Example 1

$3x^2 - 9x - 12$ (the GCF is 3)

$= 3(x^2 - 3x - 4)$ factor

$= 3(x-4)(x+1)$

The answer can be written either

$3(x-4)\ (x+1)$

or

$3(x+1)\ (x-4)$

Example 2

$2x^2 - 18x + 36$ (the GCF is 2)

$= 2(x^2 - 9x + 18)$ (factor)

$= 2(x-6)(x-3)$

The answer can be written either

$2(x-6)\ (x-3)$

or

$2(x-3)\ (x-6)$

Example 3

$4x^2 - 4x - 80$ (the GCF is 4)

$= 4(x^2 - x - 20)$ (factor)

$= 4(x-5)(x+4)$

The answer can be written either

$4(x-5)\ (x+4)$

or

$4(x+4)\ (x-5)$

Example 4

$5x^2 - 30x + 45$ (the GCF is 5)

$= 5(x^2 - 6x + 9)$ (factor)

$= 5(x-3)(x-3)$

Example 5

$5x^2 - 10x + 20$ (the GCF is 4)

$= 5(x^2 - 2x + 4)$

Can't be factored further

$= 5(x^2 - 2x + 4)$

Example 6

$x^2 - 7x + 5$

There is no GCF no and

the trinomial does not factor

so this DOES NOT FACTOR

Factor Completely

Factor Out The GCF

Answers will look like **distributive** problems

$10x^3 - 25x^2 + 15x$
$= 5x(2x^2 - 5x + 3)$

$27x^3 - 9x^2$
$= 9x^2(3x - 1)$

$4x^2y - 18y$
$= 2y(2x^2 - 9)$

Second Degree Trinomials $1x^2 \pm Bx \pm C$

with a Positive Last term $1x^2 \pm Bx + C$

with a Negitive Last term $1x^2 \pm Bx - C$

Trinomials with a POSITIVE LAST term

$1x^2 \pm Bx + C$
$= (x \pm \text{last}_1)(x \pm \text{last}_2)$

the 2 last terms MULTIPLY to C
the 2 last terms ADD to B

$7 \bullet 2$
$14 \bullet 1$

$x^2 - 15x + 14$
the 2 last terms multiply to 14
the 2 last terms add to -15
$= (x - 14)(x - 1)$

$3 \bullet 4$
$6 \bullet 2$
$12 \bullet 1$

$x^2 + 8x + 12$
the 2 last terms multiply to 12
the 2 last terms add to $+8$
$= (x + 6)(x + 2)$

Trinomials with a NEGITIVE LAST term

$1x^2 \pm Bx - C$
$= (x \pm \text{last}_1)(x \pm \text{last}_2)$

the 2 last terms MULTIPLY to C
the 2 last terms SUBTRACT to B

$2 \bullet 5$
$10 \bullet 1$

$x^2 - 3x - 10$
the 2 last terms multiply to -10
the 2 last terms subtract to -3
$= (x - 5)(x + 2)$

$2 \bullet 9$
$3 \bullet 6$
$18 \bullet 1$

$x^2 - 17x - 18$
the 2 last terms multiply to -18
the 2 last terms subtract to -17
$= (x - 18)(x + 1)$

Second Degree Binomials Perfect Squares

The Difference of Two Perfect Squares

$a^2 - b^2$
$= (a + b)(a - b)$

$4x^2 - 49$
$= (2x + 7)(2x - 7)$

$36x^2 - 25$
$= (6x + 5)(6x - 5)$

The Sum of Two Perfect Squares Does Not Factor

$x^2 + 4$
does not factor

$4x^2 + 25$
Prime

$16x^2 + 1$
DNF

Additional Notes

to help explain why the factoring rules we use work.

To understand how to factor $1x^2 + Bx + C$ into $(x + D)(x + E)$

we need to start with multiplying

$(x + D)(x + E)$ by the FOIL process

to see how it is related to the trinomial $1x^2 + Bx + C$

$$\begin{array}{ccccc} & \text{F} & \text{O} & \text{I} & \text{L} \\ (x+D)\,(x+E) = & x \bullet x & + \, Dx & + \, Ex & + \, D \bullet E \end{array}$$

$$\downarrow \qquad \underbrace{Dx + Ex}_{\downarrow} \qquad \downarrow$$

$$= x^2 + (D+E)x + D \bullet E$$

$$(x+D)\,(x+E) = \underset{\text{first}}{x^2} + \underset{\text{middle}}{(D+E)x} + \underset{\text{last}}{D \bullet E}$$

If $x^2 + Bx + C$ must equal $(x + D)(x + E)$

then

$x^2 \quad + Bx \quad + C$

$= x^2 + (D + E)x + DE$

and D and E must

multiply to give + C

and

add to give + B

To understand how to factor $1x^2 - Bx + C$ into $(x - D)(x - E)$

we need to start with multiplying

$(x - D)(x - E)$ by the FOIL process

to see how it is related to the trinomial $1x^2 - Bx + C$

$$\begin{aligned}
& \quad\;\; \text{F} \quad \text{O} \quad \text{I} \quad \text{L} \\
(x-D)(x-E) &= x \bullet x - Dx - Ex + D \bullet E \\
& \quad \downarrow \qquad \underbrace{-Dx - Ex}_{\downarrow} \qquad \downarrow \\
&= x^2 + (-D - E)x + D \bullet E \\
(x-D)(x-E) &= x^2 + (-D - E)x + D \bullet E \\
& \quad \text{first} \qquad \text{middle} \qquad \text{last}
\end{aligned}$$

If $x^2 - Bx + C$ must equal $(x - D)(x - E)$

then

$x^2 \quad - Bx \quad + C$

$= x^2 + (-D - E)x + DE$

and D and E must

multiply to give + C

and

add to give – B

To understand how to factor $1x^2 - Bx - C$ into $(x + D)(x - E)$

we need to start with multiplying

$(x + D)(x - E)$ by the FOIL process

to see how it is related to the trinomial $1x^2 - Bx - C$

$$\begin{aligned} & \quad\ \text{F} \quad\ \text{O} \quad \text{I} \quad \text{L} \\ (x+D)\ (x-E) &= x \bullet x + Dx + Ex + (D)(-E) \\ & \downarrow \qquad \underbrace{Dx - Ex}_{\downarrow} \qquad \downarrow \\ &= x^2 + (D-E)x - D \bullet E \\ (x+D)\ (x-E) &= x^2 + (D-E)x - D \bullet E \\ & \text{first} \quad \text{middle} \quad \text{last} \end{aligned}$$

If $x^2 - Bx - C$ must equal $(x + D)(x - E)$

then

$x^2 \quad - Bx \quad - C$

$= x^2 + (D - E)x - DE$

and D and E must

multiply to give – C
and
subtract to give – B

If D and E must multiply to give – C (a negative number) then one of the numbers D or E must be positive and the other negative

If D and E must subtract to give the middle term Bx then

If the middle term is negative then D – E must be negative
If the middle term is Positive then D – E must be positive