Section 7 – 2: Factoring Perfect Squared Binomials

What is a Perfect Square?

When a positive number is **multiplied by itself** the number obtained is called a **Perfect Square**.

$1 \bullet 1 =$ 1 $2 \bullet 2 =$ 4 $3 \bullet 3 =$ 9 $4 \bullet 4 =$ 16 $5 \bullet 5 =$ 25 $6 \bullet 6 =$ 36

$7 \bullet 7 =$ 49 $8 \bullet 8 =$ 64 $9 \bullet 9 =$ 81 $10 \bullet 10 =$ 100 $11 \bullet 11 =$ 121 $12 \bullet 12 =$ 144

When a variable term is **multiplied by itself** the product is called a **perfect squared variable**.

$x \bullet x = x^2$ $y \bullet y = y^2$ $xy \bullet xy = x^2y^2$ $x^2 \bullet x^2 = x^4$ $x^3 \bullet x^3 = x^6$

The Difference of Two Perfect Squares

A special **binomial** (2 terms) called **The Difference of Two Perfect Squares** can be formed by taking any two perfect squared terms and **subtracting** them. If the 2 terms are **added** or if either term is **not a perfect square** then the Binomial is **NOT the Difference of Two Perfect Squares.**

x^2 — 9 x^2 — 4 x^2 — 81

16 x^2 — 25 64 x^2 — 100 4 x^2 — 81

Examples: Which of the following binomials are **The Difference of Two Perfect Squares?**

1. $x^2 - 9$ Yes
2. $4x^2 + 25$ No
3. $16x^2 - 49$ Yes

5. $81x^2 - 100$ Yes
6. $9x^2 - 18$ No
7. $27x^2 + 36$ No

9. $25x^2 - 11$ No
10. $25x^4 - 121$ Yes
11. $49x^6 - 144$ Yes

Factoring The Difference of Two Perfect Squares

The FOIL process changes a product of 2 binomials into a polynomial. The reverse method starts with a polynomial and finds the 2 binomials whose product will be that polynomial. The process of changing a polynomial into a product of 2 binomials is called **factoring a polynomial**. The first type of polynomial we will learn to factor is T**he Difference of Two Perfect Squares**.

When we **FOIL** the 2 binomials $(a+b)\,(a-b)$ we see that

$$(a+b)\,(a-b)$$
$$= a^2 - ab + ab - b^2$$
$$= a^2 - b^2$$

Reversing this rule gives $a^2 - b^2 = (a+b)\,(a-b)$

We call the binomial expression $a^2 - b^2$ the **Difference** of **Two Perfect Squares**

and we call the product $(a+b)\,(a-b)$ **the factored form of** $a^2 - b^2$

Factoring The Difference of Two Perfect Squares

This rule states that the factors of The Difference of Two Perfect Squares are
the product of the sum and difference of the factors
that make up each of the perfect squares.

1. Factor $x^2 - 25$

We know $x^2 = x \bullet x$ and $25 = 5 \bullet 5$

$$\begin{array}{l} a^2 - b^2 = (a + b)\,(a - b) \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ x^2 - 25 = (x + 5)\,(x - 5) \end{array}$$

The answer is $(x+5)\,(x-5)$

2. Factor $9x^2 - 16$

We know $9x^2 = 3x \bullet 3x$ and $16 = 4 \bullet 4$

$$\begin{array}{l} a^2 - b^2 = (a + b)\,(a - b) \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 9x^2 - 16 = (3x + 4)\,(3x - 4) \end{array}$$

The answer is $(3x+4)\,(3x-4)$

3. Factor $81y^2 - 49$

$$\begin{array}{l} a^2 - b^2 = (a + b)\,(a - b) \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 81y^2 - 49 = (9y + 7)(9y - 7) \end{array}$$

The answer is $(9y+7)\,(9y-7)$

4. Factor $4x^2 - y^2$

$$\begin{array}{l} a^2 - b^2 = (a + b)\,(a - b) \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 36x^2 - y^2 = (6x + y)\,(6x - y) \end{array}$$

The answer is $(6x+y)\,(6x-y)$

NOTE: The order of the factors does not matter so either $(6x+1)\,(6x-1)$ or $(6x-1)\,(6x+1)$ are OK.

NOTE: This process only works for The Difference of Two Perfect Squares. The **Sum of Two Perfect Squares does not factor**. We write **DNF** or **Prime** if a polynomial does not factor.

Factoring The Difference of Two Perfect Squares

Example 1

$x^2 - 100$
$= (x+10)(x-10)$

Example 2

$x^2 - 64$
$= (x+8)(x-8)$

Example 3

$4x^2 - 25$
$= (2x+5)(2x-5)$

Example 4

$9x^2 - 49$
$= (3x+7)(3x-7)$

Example 5

$x^2 - 25y^2$
$= (x+5y)(x-5y)$

Example 6

$4x^2y^2 - 25$
$= (2xy+5)(2xy-5)$

Factoring The Sum of Two Perfect Squares

The **Sum of Two Perfect Squares** **DOES NOT FACTOR**. Write **DNF** or **Prime** if a polynomial does not factor.

Example 9

$x^2 + 16$
Does not factor

Example 10

$9x^2 + 16$
Prime

Example 11

$4x^2 + 25$
DNF

Factoring a Polynomial Completely

To factor any polynomial completely you should **factor the GCF out FIRST** if the polynomial has a GCF. If the polynomial **remaining inside the parentheses** is the difference of two perfect squares then it must be factored using the difference of squares technique. The complete factored answer is the product of the GCF and the factored difference of squares.

Factoring The Difference of Two Perfect Squares that have a GCF

1. Factor out the GCF if there is one
2. If the remaining polynomial is the difference of two perfect squares then factor that polynomial.
   Remember the Sum of Two Perfect Squares does not factor.
3. The final answer is the product of the GCF and the factored difference of squares

Factor Completely

Remember to factor out the GCF first if there is one.

Remember the Sum of Two perfect squares does not factor

Example 1

$3x^2 - 12$ (the GCF is 3)

$= 3(x^2 - 4)$ Diff. of Squares

$= 3(x+2)(x-2)$

The answer can be written either

$3(x+2)\ (x-2)$

or

$3(x-2)\ (x+2)$

Example 2

$20x^2 - 45$ (the GCF is 5)

$= 5(4x^2 - 9)$ Diff. of Squares

$= 5(2x+3)(2x-3)$

The answer can be written either

$5(2x+3)\ (2x-3)$

or

$5(2x-3)\ (2x+3)$

Example 3

$4x^2 - 100$ (the GCF is 4)

$= 4(x^2 - 25)$ Diff. of Squares

$= 4(x+5)(x-5)$

The answer can be written either

$4(x+5)\ (x-5)$

or

$4(x-5)\ (x+5)$

Example 4

$49x^4 - 36x^2$ (the GCF is x^2)

$= x^2(49x^2 - 36)$ Diff. of Squares

$= x^2(7x-6)(7x+6)$

Example 5

$2x^2 + 18$ (the GCF is 2)

$= 2(x^2 + 9)$ Can't factor further

$= 2(x^2 + 9)$

Example 6

$3x^2 + 5$

There is no GCF and

no diference of two squares

so this DOES NOT FACTOR

Example 7

$2x^2 - 50$

the GCF is 2

$= 2\left(x^2 - 25\right)$

Diff. of Squares

$= 2(x+5)(x-5)$

Example 8

$3x^3 - 12x$

the GCF is $3x$

$= 3x\left(x^2 - 4\right)$

Diff. of Squares

$= 3x(x-2)(x+2)$

Example 9

$2x^2 + 18$

ther GCF is 2

$= 2\left(x^2 + 9\right)$

not the Dif. of Squares

$= 2\left(x^2 + 9\right)$

Factoring out a Negative One

The normal order used to write a binomial that is the Difference of Two Perfect Squares for the x^2 term to have a **positive coefficient** and be written first. The **constant term is written last** and is **negative**. An example of this is $x^2 - 9$

Sometime the Difference of Two Perfect Squares is written with the constant term being positive and written first and the x^2 term is negative and written last. An example of this is $9 - x^2$

We could factor $9 - x^2$ as $(3 - x)(3 + x)$ but for the factoring problems in this chapter we want to preserve the order of writing the x^2 as a positive term and writing it first and the constant term being negative and written second. To write $9 - x^2$ in that order we will **factor out a** -1 from each term in the expression. This will put a -1 outside a parentheses and leave a $x^2 - 9$ inside. We can then factor the remaining expression inside the parenthesis as before.

Example 1

$9 - x^2$

factor out a -1

$-1(x^2 - 9)$

factor the $x^2 - 9$

$-(x - 3)\ (x + 3)$

Example 2

$16 - x^2$

factor out a -1

$-1(x^2 - 16)$

factor the $x^2 - 16$

$-(x - 4)\ (x + 4)$

Example 3

$49 - x^2$

factor out a -1

$-1(x^2 - 49)$

factor the $x^2 - 49$

$-(x - 7)\ (x + 7)$

Example 4

$25 - 4x^2$

factor out a -1

$-1(4x^2 - 25)$

factor the $4x^2 - 25$

$-(2x - 5)\ (2x + 5)$

Example 5

$1 - 9x^2$

factor out a -1

$-1(9x^2 - 1)$

factor the $9x^2 - 1$

$-(3x - 1)\ (3x + 1)$

Example 6

$9 - 16x^2$

factor out a -1

$-1(16x^2 - 9)$

factor the $16x^2 - 9$

$-(4x - 3)\ (4x + 3)$

