

## Section 6 – 2A:

## Scientific Notation

A common use of positive and negative exponents is writing numbers in scientific notation. In astronomy, the distance between 2 objects can be very large and the numbers often contain many zeros.

The distance from the Sun to the Earth is 93,000,000 miles.  
The digits 9 and 3 are important but all the zeros are just place holders.

The wavelength of infrared light is .000000037 meters.  
The digits 3 and 7 are important but all the zeros are just place holders.

Scientific notation is most useful in writing very large numbers that end in many zeros or decimal numbers less than 1 that start with several zeros. In both cases the zeros act as place holders. The use of scientific notation allows us to keep the important digits but avoid writing all the zeros.

520,000,000 can be written as  $5.2 \times 10^8$  .0000000083 can be written as  $8.3 \times 10^{-9}$

**A number is said to have been written in Scientific Notation if it is in the form of a number written in the form  $A \times 10^n$  where **A** is a number between zero and 10 and **n** is the exponent of 10 and is a positive or negative integer.**

$5.2 \times 10^4$                        $9.83 \times 10^{15}$                        $7.25 \times 10^{-17}$                        $3.47 \times 10^{-12}$

1. The number to the **left of the decimal** is a digit from 1 to 9.
2. The decimal number is multiplied by to to some exponent.
3. The exponent of 10 is a positive or negative integer.

### Converting a Number in Scientific Notation Into A Decimal Number

#### If The Power Of Ten Is Positive

A positive exponent above the 10 means we are **multiplying** the decimal number by ten the number of times as the tens exponent. **When you multiply a number by ten the decimal point is moved to the RIGHT one place for each time you multiply by 10.**

#### Example 1

$$5.2 \times 10^6$$

means

$$5.2 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10$$

We multiply 5.2 by ten 6 times.

The decimal point is moved

6 places **to the right**

$$5.2 \times 10^6 = 5,200,000$$

#### Example 2

$$9.32 \times 10^5$$

means

$$9.32 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10$$

We multiply 9.32 by ten 5 times.

The decimal point is moved

5 places **to the right.**

$$9.32 \times 10^5 = 932,000$$

## Converting a Number in Scientific Notation Into A Decimal Number If The Power Of Ten Is Negative

A negative exponent means we are dividing the **decimal** number by ten the number of times as the tens exponent. When you divide by ten **the decimal point is moved to the LEFT one place for each time you multiply by 10.**

### Example 1

$$7.63 \times 10^{-5}$$

means

$$7.63 \div 10 \div 10 \div 10 \div 10 \div 10$$

We 7.63 divide by ten 4 times.

The decimal point is moved  
5 places **to the left**

$$7.6 \times 10^4 = .00076$$

### Example 2

$$9.21 \times 10^{-6}$$

means

$$9.21 \div 10 \div 10 \div 10 \div 10 \div 10 \div 10$$

We divide 9.21 by ten 5 times.

The decimal point is moved  
6 places **to the left.**

$$9.21 \times 10^{-6} = .00000921$$

## Converting a Number in Scientific Notation Into A Decimal Number

To convert a number in the form  $A \times 10^n$  Into A Decimal Number

1. **If n is positive** then move the decimal point to the **RIGHT** n places

### Example 3

$$5.2 \times 10^4$$

move the decimal point

4 places to the right

$$= 52,000$$

### Example 4

$$8.24 \times 10^6$$

move the decimal point

6 places to the right

$$= 8,240,000$$

### Example 5

$$3.046 \times 10^8$$

move the decimal point

8 places to the right

$$= 304,600,000$$

2. **If n is negative** then move the decimal point to the **LEFT** n places

### Example 6

$$4.9 \times 10^{-4}$$

move the decimal point

4 places to the left

$$= .00049$$

### Example 7

$$3.97 \times 10^{-6}$$

move the decimal point

6 places to the left

$$= .00000397$$

### Example 8

$$2.01 \times 10^{-7}$$

move the decimal point

7 places to the left

$$= .000000201$$

## Converting Decimals Numbers Into Scientific Notation Form

We can reduce the number of zeros in many decimal numbers by moving the decimal until there is only **1 digit to the left of the decimal** and then showing how many places the decimal was moved as an exponent of 10.

Step 1. Move the decimal place so that there is **ONE** nonzero digit to the **left of the decimal**.

Step 2. Count the number of places you moved the decimal and notice the direction it was moved.

Step 3. Write the new number with only the nonzero digits times 10 to a power.

3A. If you **moved the decimal to the right** the exponent above the 10 will be a **positive number** equal to the number of places the decimal points was moved to the right.

3B. If you **moved the decimal to the left** the exponent above the 10 will be a **negative number** equal to the number of places the decimal points was moved to the left.

**For decimals numbers greater than one**  
the decimal is moved to the left and a **positive exponent** is put above the 10

### Example 1

9,300,000

9300000

the decimal was moved

6 places to the left

the exponent above

10 is +6

$$9,300,000 = 9.3 \times 10^6$$

### Example 2

434,000,000

434000000

the decimal was moved

8 places to the left

the exponent above

10 is +8

$$434,000,000 = 4.34 \times 10^8$$

### Example 3

35,700,000

35700000

the decimal was moved

7 places to the left

the exponent above

10 is +7

$$35,700,000 = 3.57 \times 10^7$$

## For decimals numbers less than one

The decimal is moved to the right and a **negative exponent** is put above the 10

### Example 4

.000078

00007.8

the decimal was moved

5 places to the right

the exponent above

10 is -5

$$.000078 = 7.8 \times 10^{-5}$$

### Example 5

.000629

0006.29

the decimal was moved

4 places to the right

the exponent above

10 is -4

$$.000629 = 6.29 \times 10^{-4}$$

### Example 6

.0000081

000008.1

the decimal was moved

6 places to the right

the exponent above

10 is -6

$$.0000081 = 8.1 \times 10^{-6}$$

## Multiplying Numbers in Scientific Notation Form

Two numbers in Scientific Notation can be multiplied using the product rule.

$$\begin{aligned}(A \times 10^n)(B \times 10^m) \\ &= A \cdot B \times 10^n \times 10^m \\ &= A \cdot B \times 10^{n+m}\end{aligned}$$

### Example 1

$$\begin{aligned}(1.5 \times 10^5)(3 \times 10^6) \\ &= 1.5 \times 3 \times 10^5 \times 10^6 \\ &= 4.5 \times 10^{5+6} \text{ (product rule )} \\ &= 4.5 \times 10^{11}\end{aligned}$$

### Example 2

$$\begin{aligned}(3 \times 10^4)(5 \times 10^2) \\ &= 3 \times 5 \times 10^4 \times 10^2 \\ &= 15 \times 10^{4+2} \text{ (product rule )} \\ &= 15 \times 10^6 \\ &= 1.5 \times 10^1 \times 10^6 \\ &= 1.5 \times 10^7\end{aligned}$$

### Example 3

$$\begin{aligned}(3.2 \times 10^2)(7 \times 10^{-11}) \\ &= 3.2 \times 7 \times 10^2 \times 10^{-11} \\ &= 22.4 \times 10^{2-11} \text{ (product rule )} \\ &= 22.4 \times 10^{-9} \\ &= 2.24 \times 10^1 \times 10^{-9} \\ &= 2.4 \times 10^{-8}\end{aligned}$$

Very large numbers that end with several zeros or very small numbers that have zeros to the left of the decimal point can be converted into scientific notation and then multiplied as shown below.

### Example 4

$$\begin{aligned}(5000)(.000003) \\ &= 5 \times 10^3 \times 3 \times 10^{-6} \\ &= 5 \times 3 \times 10^3 \times 10^{-6} \\ &= 15 \times 10^{-3} \\ &= 15 \times 10^1 \times 10^{-3} \\ &= 1.5 \times 10^{-2}\end{aligned}$$

### Example 5

$$\begin{aligned}(.0006)(.03) \\ &= 6 \times 10^{-4} \times 3 \times 10^{-2} \\ &= 6 \times 3 \times 10^{-4} \times 10^{-2} \\ &= 18 \times 10^{-6} \\ &= 1.8 \times 10^1 \times 10^{-6} \\ &= 1.8 \times 10^{-5}\end{aligned}$$

### Example 6

$$\begin{aligned}(.00005)(.071) \\ &= 5 \times 10^{-5} \times 7.1 \times 10^{-2} \\ &= 5 \times 7.1 \times 10^{-5} \times 10^{-2} \\ &= 35.5 \times 10^{-7} \\ &= 3.55 \times 10^1 \times 10^{-7} \\ &= 3.5 \times 10^{-6}\end{aligned}$$

## Dividing Numbers in Scientific Notation Form

Two numbers in Scientific Notation can be divided by using the quotient rule

$$\frac{A \times 10^n}{B \times 10^m} = \frac{A}{B} \cdot \frac{10^n}{10^m}$$

### Example 7

$$\frac{8 \times 10^7}{2 \times 10^4}$$

$$= \frac{8}{2} \cdot \frac{10^7}{10^4}$$

(use the quotient rule)

$$= \frac{8}{2} \cdot 10^{7-4}$$

$$= 4 \times 10^3$$

### Example 8

$$\frac{4.5 \times 10^9}{3 \times 10^2}$$

$$= \frac{4.5}{3} \cdot \frac{10^9}{10^2}$$

(use the quotient rule)

$$= \frac{4.5}{3} \cdot 10^{9-2}$$

$$= 1.5 \times 10^7$$

### Example 9

$$\frac{6.3 \times 10^8}{3 \times 10^3}$$

$$= \frac{6.3}{3} \cdot \frac{10^8}{10^3}$$

(use the quotient rule)

$$= \frac{6.3}{3} \cdot 10^{8-3}$$

$$= 2.1 \times 10^5$$

Very large numbers that end with several zeros or very small numbers that have zeros to the left of the decimal point can be converted into scientific notation and then divided as shown below.

### Example 10

$$\frac{16000000 \cdot .003}{240}$$

$$= \frac{1.6 \times 10^7 \times 3 \times 10^{-3}}{2.4 \times 10^2}$$

$$= \frac{1.6 \cdot 3}{2.4} \cdot \frac{10^7 \cdot 10^{-3}}{10^2}$$

$$= 2 \times 10^2$$

### Example 11

$$\frac{120000 \cdot .02}{.00004}$$

$$= \frac{1.2 \times 10^5 \times 2 \times 10^{-2}}{4 \times 10^{-5}}$$

$$= \frac{1.2 \cdot 2}{4} \cdot \frac{10^5 \cdot 10^{-2}}{10^{-5}}$$

$$= .6 \times 10^8$$

$$= 6 \times 10^7$$

### Example 12

$$\frac{.0000006 \cdot .08}{1200000}$$

$$= \frac{6 \times 10^{-7} \times 8 \times 10^{-2}}{12 \times 10^6}$$

$$= \frac{6 \cdot 8}{12} \cdot \frac{10^{-7} \cdot 10^{-2}}{10^6}$$

$$= 4 \times 10^{-3}$$

## Adding or Subtracting Numbers in Scientific Notation

NOTE: The rules used for multiplying and dividing numbers in scientific notation **CANNOT** be used for the addition or subtraction of numbers in scientific notation. To add or subtract numbers in scientific notation the powers of ten for each term **must be the same**.

You can only add or subtract A and B if both powers of ten are the same

$$A \times 10^n + B \times 10^n = A + B \times 10^n$$

and

$$A \times 10^n - B \times 10^n = A - B \times 10^n$$

### Example 13

$$5.2 \times 10^4 + 3.4 \times 10^4$$

both terms have the same power of 10 ( $10^4$ )

$$= 5.2 + 3.4 \times 10^4$$

$$= 8.6 \times 10^4$$

### Example 14

$$8.7 \times 10^{-5} - 2.4 \times 10^{-5}$$

both terms have the same power of 10 ( $10^{-5}$ )

$$= 8.7 - 2.4 \times 10^{-5}$$

$$= 6.3 \times 10^{-5}$$

### Example 15

$$7.1 \times 10^5 + 5.6 \times 10^5$$

both terms have the same power of 10 ( $10^5$ )

$$= 7.1 + 5.6 \times 10^5$$

$$= 12.7 \times 10^5$$

$$= 1.27 \times 10^6$$

### Example 16

$$9.1 \times 10^{-8} - 6.5 \times 10^{-8}$$

both terms have the same power of 10 ( $10^{-8}$ )

$$= 9.1 - 6.5 \times 10^{-8}$$

$$= 2.6 \times 10^{-8}$$

## Estimating a Number Using Scientific Notation

It is often useful to round off very large numbers or very small numbers and then put the rounded off number into scientific notation.

### Example 17

8,234  
can be rounded off to  
8,000  
and expressed as  
 $8 \times 10^3$

### Example 18

42,234  
can be rounded off to  
40,000  
and expressed as  
 $4 \times 10^4$

### Example 19

5,678,234  
can be rounded off to  
5,000,000  
and expressed as  
 $5 \times 10^6$

### Example 20

.00686  
can be rounded off to  
.007  
and expressed as  
 $7 \times 10^{-3}$

### Example 21

.00024  
can be rounded off to  
.0002  
and expressed as  
 $2 \times 10^{-4}$

### Example 22

.0000276  
can be rounded off to  
.00003  
and expressed as  
 $3 \times 10^{-5}$

## Estimating an Expression Using Scientific Notation

It is most common to use a calculator to perform the following calculations. If an estimate is required instead of the exact answer then rounding off each number and then putting that number into scientific notation allows us to get an estimate using the rules for exponents.

### Example 23

Estimate  
$$\frac{39765 \cdot .00345}{215}$$
round off each number  
and put that number into  
Scientific Notation  
$$\cong \frac{4 \times 10^4 \times 3 \times 10^{-3}}{2 \times 10^2}$$
and then use the exponent  
rules to simplify  
$$\cong \frac{4 \cdot 3 \cdot 10^4 \cdot 10^{-3}}{2 \cdot 10^2}$$
$$\cong 6 \times 10^{-1}$$
$$\cong .6$$

### Example 24

Estimate  
$$\frac{8199 \cdot 891000}{59000}$$
round off each number  
and put that number into  
Scientific Notation  
$$\cong \frac{8 \times 10^3 \times 9 \times 10^5}{6 \times 10^4}$$
and then use the exponent  
rules to simplify  
$$\cong \frac{8 \cdot 9 \cdot 10^3 \cdot 10^5}{6 \cdot 10^4}$$
$$\cong 12 \times 10^4$$
$$\cong 120,000$$

### Example 25

Estimate  
$$\frac{219999 \cdot .000599}{.000061}$$
round off each number  
and put that number into  
Scientific Notation  
$$\cong \frac{2 \times 10^5 \times 6 \times 10^{-4}}{6 \times 10^{-5}}$$
and then use the exponent  
rules to simplify  
$$\cong \frac{2 \cdot 9 \cdot 10^5 \cdot 10^{-4}}{6 \cdot 10^{-5}}$$
$$\cong 3 \times 10^6$$
$$\cong 3,000,000$$