Section 6 – 2: Negative Exponents

A Variable with a Negative Exponent

Changing a Negative Exponent into Positive Exponent.

If x is any non zero number then $x^{-1} = \frac{1}{x}$ and $\frac{1}{x^{-1}} = x$

If a base has a **negative exponent then it must be moved.** If the base with the **negative exponent** is in the numerator (top) of a fraction it must be moved to the denominator (bottom) and the exponent made positive. If the base with the **negative exponent** is in the denominator (bottom) of a fraction it must be moved to the numerator (top) and the exponent made positive.

Example 1

x^{-2}

the base of x has a negitive exponent

x^{-2} must be moved to the bottom

and the exponent made positive

$$x^{-2} = \frac{1}{x^2}$$

Example 2

$$\frac{y^{-4}}{x^2}$$

the base of y has a negitive exponent

y^{-4} must be moved to the bottom

and the exponent made positive

$$\frac{y^{-4}}{x^2} = \frac{1}{x^2 y^4}$$

Example 3

$$\frac{1}{x^{-4}}$$

the base of x has a negitive exponent

x^{-4} must be moved to the top

and the exponent made positive

$$\frac{1}{x^{-4}} = x^4$$

Example 4

$$\frac{1}{x^{-5} y^3}$$

the base of x has a negitive exponent

x^{-5} must be moved to the top

and the exponent made positive

$$\frac{1}{x^{-5} y^3} = \frac{x^5}{y^3}$$

Example 5

$$\frac{y^{-3}}{x^{-2}}$$

y^{-3} must move to the bottom

x^{-2} must move to the top

$$\frac{y^{-3}}{x^{-2}} = \frac{x^2}{y^3}$$

Example 6

$x^{-2}y^{-3}$

y^{-3} must move to the bottom

x^{-2} must move to the bottom

$$x^{-2}y^{-3} = \frac{1}{x^2 y^3}$$

A Constant with a Negative Exponent

A Constant with **a negative exponent** is treated just like a variable with a negative exponent.

A constant with a negative number in front **DOES NOT MOVE.**

Example 7

3^{-2}

the base of 3 has a negitive exponent

3^{-2} must be moved to the bottom and the exponent made positive

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

Example 8

$$\frac{1}{2^{-3}}$$

the base of 2 has a negitive exponent

2^{-3} must be moved to the top and the exponent made positive

$$\frac{1}{2^{-3}} = 2^3 = 8$$

Example 9

$$\frac{3^{-2}}{4}$$

the base of 3 has a negitive exponent

3^{-2} must be moved to the bottom and the exponent made positive

$$\frac{3^{-2}}{4} = \frac{1}{4 \bullet 3^2} = \frac{1}{36}$$

Example 10

$$\frac{2^{-3}}{5^{-2}}$$

2^{-3} must move to the bottom

5^{-2} must move to the top

$$\frac{2^{-3}}{5^{-2}} = \frac{5^2}{2^3} = \frac{25}{8}$$

Sample Problems

Simplify. Leave no negative exponents in your answer

Example 11

$$\frac{-3x}{2x^{-4}}$$

$$= \frac{-3x \bullet x^4}{2}$$

$$= \frac{-3x^5}{2}$$

Example 12

$$\frac{2^{-3}y^{-2}}{y^6}$$

$$= \frac{1}{2^3 y^6 \bullet y^2}$$

$$= \frac{1}{8y^8}$$

Example 13

$$\frac{3y^{-7}}{4^{-1}x^2}$$

$$\frac{3 \bullet 4}{y^7 x^2}$$

$$\frac{12}{y^7 x^2}$$

Example 14

$$\frac{3^{-2}y^{-5}}{x^{-4}}$$

$$\frac{x^4}{3^2 y^5}$$

$$= \frac{x^4}{9y^5}$$

Power Rule, Product Rule, Negative Exponents, Quotient Rule

Putting them all together

The order of operations **PEMDAS** requires that we **perform the Power Rule first.** After the power rule has been performed there are several options for the order of the remaining rules to be performed. A common order is listed below.

Step 1: **The Power Rule must be done first.** This is a requirement of PEMDAS

Step 2: Perform the **Product Rule** if there is a product of common variables.

Step 3: Move a base with a **Negative Exponent.**

Step 4: Perform **the Quotient Rule** If there is a common variable on the **top and bottom** of the fraction.

Some students will move a base with a negative exponent second. They will then use what ever combination of product and quotient rules are needed to finish the process. Either of the suggested orders will work but **the Power Rule must always be performed first.**

Example 1

Step 1. Negative Exponents
Step 2. Quotient Rule

$$\frac{x^{-7}y^{-4}}{x^{-4}y^{-9}}$$

move the bases with negitive exponents

$$=\frac{x^4y^9}{x^7y^4}$$

use the quotient rule for

$$\frac{x^4}{x^7} \text{ and } \frac{y^9}{y^4}$$

$$=\frac{y^5}{x^3}$$

Example 2

Step 1. Negative Exponents
Step 2. Quotient Rule

$$\frac{x^5y^{-1}}{3^{-2}x^6y^{-7}}$$

move the bases with negitive exponents

$$=\frac{3^2x^5y^7}{x^6y}$$

use the quotient rule for

$$\frac{x^5}{x^6} \text{ and } \frac{y^7}{y}$$

$$=\frac{9y^6}{x}$$

Example 3

1. Power Rule
2. Negative Exponents

$$\left(2xy^{-2}\right)^{-3}$$

perform the Power Rule by
multipling each exponent inside
by the exponent outside

$$\left(2^1x^1y^{-2}\right)^{-3}$$
$$=2^{-3}x^{-3}y^{6}$$

move the bases with negitive exponents

$$=\frac{y^6}{8x^3}$$

Example 4

1. Power Rule
2. Negative Exponents

$$\frac{1}{\left(3x^2y^{-3}\right)^{-2}}$$

perform the Power Rule by
multipling each exponent inside
by the exponent outside

$$\frac{1}{\left(3^1x^2y^{-3}\right)^{-2}}$$

$$\frac{1}{3^{-2}x^{-4}y^6}$$

move the bases with negitive exponents

$$=\frac{9x^4}{y^6}$$

Example 5

1. Product Rule
2. Negative Exponents

$$\left(5x^{-2}y^5\right)\left(-3x^{-5}y^{-2}\right)$$

perform the Product Rule

$$-15x^{-2-5}y^{5-2}$$

$$=-15x^{-7}y^3$$

move the bases with negitive exponents

$$=\frac{-15y^3}{x^7}$$

Example 6

1. Product Rule
2. Negative Exponents

$$\frac{1}{\left(2x^{-3}y^{-6}\right)\left(4x^5y^{-3}\right)}$$

perform the Product Rule

$$\frac{1}{8x^{-3+5}y^{-6-3}}$$

$$\frac{1}{8x^2y^{-9}}$$

move the bases with negitive exponents

$$=\frac{y^9}{8x^2}$$

Example 7

1. Product Rule
2. Negative Exponents
3. Quotient Rule and Product Rule

$$\frac{\left(x^{-3}y^{-5}\right)\left(x^{-2}y\right)}{x^{3}y^{-7}}$$

$$= \frac{x^{-5}y^{-4}}{x^{3}y^{-7}}$$

$$= \frac{y^{7}}{x^{5}x^{3}y^{4}}$$

$$= \frac{y^{3}}{x^{8}}$$

Example 8

1. Product Rule
2. Negative Exponents
3. Quotient Rule and Product Rule

$$\frac{2x^{4}y^{-2}}{\left(3x^{8}y^{-2}\right)\left(4x^{-2}y^{-3}\right)}$$

$$= \frac{2x^{4}y^{-2}}{12x^{6}y^{-5}}$$

$$= \frac{2x^{4}y^{5}}{12x^{6}y^{2}}$$

$$= \frac{y^{3}}{6x^{2}}$$

Some students will move a base with a negative exponent second. They then use what ever combination of product and quotient rules are needed to finish the process. Either of the suggested orders will work but **the power Rule must always be performed first.**

Example 9

1. Power rule
2. Negative Exponents
3. Product Rule

$$\frac{\left(x^{2}y^{-3}\right)^{-2}}{\left(xy^{-3}\right)^{3}}$$

$$= \frac{x^{-4}y^{6}}{x^{3}y^{-9}}$$

$$= \frac{y^{6}y^{9}}{x^{4}x^{3}}$$

$$= \frac{y^{15}}{x^{7}}$$

Example 10

1. Power Rule
2. Negative Exponents
3. Quotient Rule and Product Rule

$$\frac{\left(3x^{-4}y^{-1}\right)^{2}}{\left(x^{2}y^{-3}\right)^{-3}}$$

$$= \frac{9x^{-8}y^{-2}}{x^{-6}y^{9}}$$

$$= \frac{9x^{6}}{x^{8}y^{2}y^{9}}$$

$$= \frac{9}{x^{2}y^{11}}$$

