

Section 5 – 4: Word Problems with Systems of Equations

In Chapter 3 we solved word problems by letting one of the unknowns be x and then stating all the other expressions in terms of x . All of the expressions were then combined into a single equation with x as the only variable. We then solved the single equation for x .

The word problems in this section can be best solved by **stating one of the unknowns as x and the other as y** . The two sentences in each problem will allow you to write **two separate equations** and **each equation will have an x and y variable**. After finding the 2 equations with x and y in them the system is then solved by either **elimination** or **substitution**.

Example 1. The **sum of two positive numbers** is 14. The **difference of the two positive numbers** is 2. Find both of the numbers.

Let one number = x
 the other number = y

Sentence	Meaning	Equation
The sum of two positive numbers is 14	If we add the two numbers ($x+y$) the sum is 14	$x + y = 14$
The difference of the two numbers is 2	If we subtract the two numbers ($x-y$) the difference is 2	$x - y = 2$

$$\begin{cases} x + y = 14 \\ x - y = 2 \end{cases} \quad \text{add both equations}$$

$$2x = 16 \quad \text{solve for } x$$

$$x = 8$$

plug $x = 8$ into either of the original equations and solve for y

$$8 + y = 14$$

$$y = 6$$

Answer: The numbers are 8 and 6

Check

$$8 + 6 = 14$$

$$8 - 6 = 2$$

Example 2. The sum of two positive numbers is 24. One number is 12 less than three times the other number. Find both of the numbers.

Let one number = x
 the other number = y .

Sentence	Meaning	Equation
The sum of two positive numbers is 24	If we add the two numbers ($x + y$) the sum is 24 = 24	$x + y = 24$
One number is 12 less than three times the other number	One number is $x =$ 12 less than three times the other number (y) $3y - 12$	$x = 3y - 12$

$$\begin{cases} x + y = 24 \\ x = 3y - 12 \end{cases} \text{ substitutue the } 3y - 12 \text{ for } x \text{ in the top equation}$$

$$\begin{aligned} (x) + y &= 24 && \text{substitutue the } 3y - 12 \text{ for } x \\ (3y - 12) + y &= 24 && \text{solve for } y \end{aligned}$$

$$\begin{aligned} 3y - 12 + y &= 24 \\ 4y - 12 &= 24 \\ 4y &= 36 \\ y &= 9 \end{aligned}$$

plug $y = 9$ into the second of the original equations and solve for x

$$\begin{aligned} x &= 3y - 12 \text{ and } y = 9 \\ x &= 3(9) - 12 \\ x &= 27 - 12 \\ x &= 15 \end{aligned}$$

Answer: The numbers are 15 and 9

Check

$$\begin{aligned} 15 + 9 &= 24 \\ 15 &= 3(9) - 12 \end{aligned}$$

Example 3. The difference of two positive numbers positive is 12. The largest number is 4 less than three times the smaller number. Find both of the numbers.

Let the largest number = x
the smaller number = y

Note: If both numbers are positive then the difference must be written as largest minus smallest to get a positive 12 as an answer

Sentence	Meaning	Equation
The difference of two positive numbers is 12	If we subtract the two numbers $x - y$ the difference is 12	$x - y = 12$
The largest number is 4 less than three times the other number	the largest number (x) is = 4 less than three times the other number (y) $3y - 4$	$x = 3y - 4$

$$\begin{cases} x - y = 12 \\ x = 3y - 4 \end{cases} \text{ substitutue the } 3y - 4 \text{ for } x \text{ in the top equation}$$

$$(x) + y = 24 \quad \text{substitutue the } 3y - 12 \text{ for } x$$

$$(3y - 4) - y = 12 \quad \text{solve for } y$$

$$3y - 4 - y = 12$$

$$2y - 4 = 12$$

$$2y = 16$$

$$y = 8$$

plug $y = 8$ into the second of the original equations and solve for x

$$x = 3y - 4 \text{ and } y = 8$$

$$x = 3(8) - 4$$

$$x = 20$$

Answer: The largest number is 20 and the smallest number is 8

Check:

$$20 - 8 = 12$$

$$20 = 3(8) - 4$$

Example 4. The total number of doors and windows in a house is 45. The number of windows is 10 more than six times the number of doors. Find the total number of windows and doors.

Let doors = D
 windows = W

Sentence	Meaning	Equation
The total number of doors and windows is 45	If we add the number of doors and windows the sum is 45	$D + W = 45$
The number of windows is 10 more than six times the number of doors	The number of windows (w) is = 10 more than six times the number of doors (d) $6D + 10$	$W = 6D + 10$

$$\begin{cases} D + W = 45 \\ W = 6D + 10 \end{cases} \text{ substitutue the } 6D + 10 \text{ for } W \text{ in the top equation}$$

$$D + (W) = 45 \quad \text{substitutue the } 6D + 10 \text{ for } W$$

$$D + (6D + 10) = 45 \quad \text{solve for } W$$

$$D + 6D + 10 = 45$$

$$7D + 10 = 45$$

$$7D = 35$$

$$D = 5$$

plug $D = 5$ into the second equation and solve for W

$$W = 6D + 10 \text{ and } D = 5$$

$$W = 6(5) + 10$$

$$W = 30 + 10$$

$$W = 40$$

ANSWER: There are 5 doors and 40 windows

Check

$$5 + 40 = 45$$

$$40 = 6(5) + 10$$

Example 5. The total number of doctors and nurses at a clinic is 11. Three times the number of doctors plus 2 times the nurses is 30. Find the total number of doctors and nurses.

Let doctors = D
 nurses = N

Sentence	Meaning	Equation
The total number of doctors and nurses is 11	If we add the number of doctors and nurses the sum is 11	$D + N = 11$
Three times the number of doctors plus 2 times the numbers of nurses is 30	three times the number of doctors (3D) plus 2 times the number of nurses (2N) is 30	$3D + 2N = 30$

$$\begin{cases} D + N = 11 \\ \underline{3D + 2N = 30} \end{cases}$$

$$-3 \begin{cases} D - N = 11 & \text{multiply this row by } -3 \\ \underline{3D + 2N = 30} \end{cases}$$

$$\begin{cases} -3D - 3N = -33 \\ \underline{3D + 2N = 30} \end{cases}$$

$$-N = -3$$

$$N = 3$$

plug $N = 3$ into $D + N = 11$ and solve for D

$$D + 3 = 11$$

$$D = 8$$

Answer: $D = 8$ and $N = 3$

ANSWER: There are 8 doctors and 3 nurses

Check

$$8 + 3 = 11$$

$$3(8) + 2(3) = 30$$