Section 4 – 5 : Finding the Equation of A Line

In the last section you were **given an equation of a line** and asked to find the slope (m) and the y intercept (b) of the line. In this section you will be given the slope and the y intercept of a line and then asked to find the equation of that line.

The type of equation a line has is based on the slope of the line.

There are three different kinds of linear equations possible
and each equation corresponds to
one of three different possible slopes.

Lines that have a zero slope

have equations of the form

y = a constant

like $y = 2$

and are graphed as a horizontal line through the y axis

Lines that have an undefined slope

have equations of the form

x = a constant

like $x = 4$

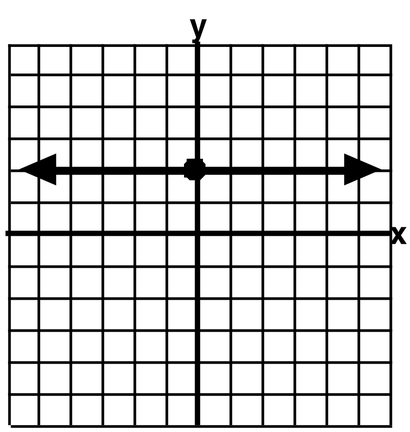
and are graphed as a vertical line through the x axis

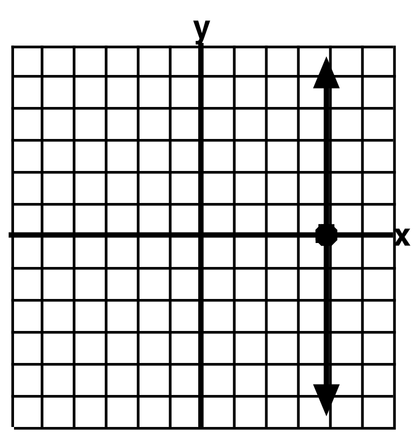
Lines that have a slope m that is a non zero number

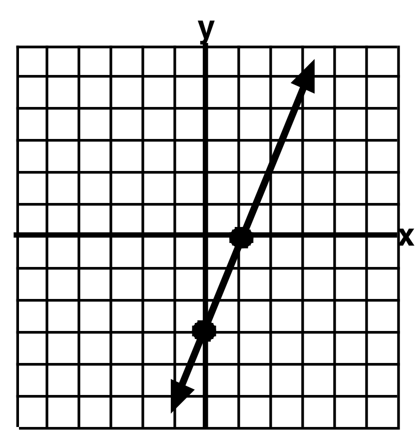
have equations of the form
$y = mx \pm b$

like
$y = 3x - 5$ or
$y = -2x + 4$

and are graphed as a sloped line through the x and y axis

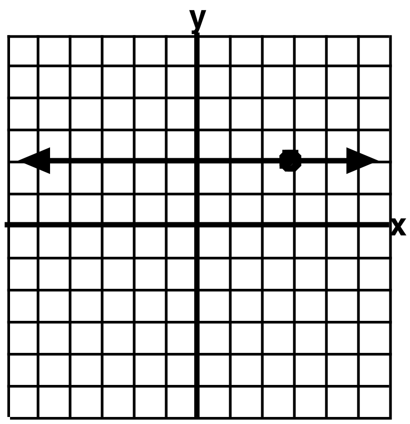






Finding the Equation Of A Line given the slope of the line m and a point (x_1, y_1) that the line passes through

Case 1: If the slope m = 0



1. The line has a slope of 0 and passes thorough (x_1, y_1)

The equation of the line is $y = y_1$

Example at left:

The line has a slope of 0 and passes thorough $(2,3)$

The equation of the line is $y = 3$

Find the equation of the line if the line has a slope m = 0 and passes through the point (x_1, y_1)

Example 1

Find the equation of the line that has a slope $m = 0$ and passes through (3,5)

$y = 5$

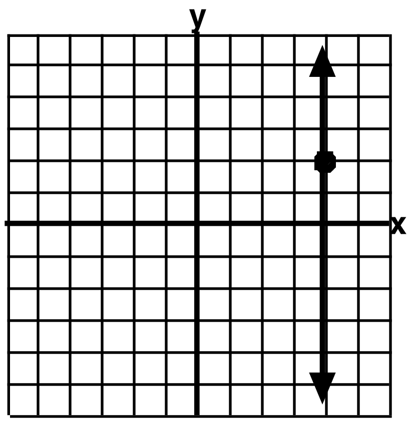
Example 2

Find the equation of the line that has a slope $m = 0$ and passes through $(4, -2)$

$y = -2$

Finding the Equation of a Line given the slope of the line and a point the line passes through

Case 2: If the slope m is undefined



2. The line's slope is undefined and goes thorough (x_1, y_1)

The equation of the line is $x = x_1$

Example at left:

The line's slope is undefined and goes thorough $(4,2)$

The equation of the line is $x = 4$

Find the equation of the line if the line has an undefined slope and passes through the point (x_1, y_1)

Example 4

Find the equation of the line that has an undefined slope and passes through $(3,5)$

$x = 3$

Example 5

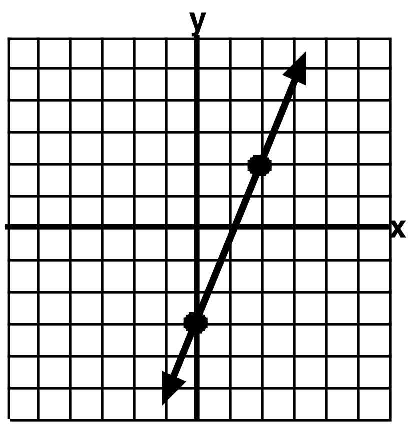
Find the equation of the line that has an undefined slope and passes through $(-1,-4)$

$x = -1$

Finding the Equation Of a Line given the slope of the line m and the y intercept $(0, b)$

If the line passes through a point that has an x coordinate of 0 then the coordinates of that point are $(0, b)$. The y coordinate of that point is b and that is the y intercept.

If the line has a slope that is a non zero number m and has a y-intercept of $(0, b)$ then The equation of the line is $y = mx + b$



Example at left:

The line's slope is $\frac{2}{5}$ **and the y-intercept is** $(0,-3)$

m= $\frac{2}{5}$ **and b= –3**

The equation of the line is $y = \frac{2}{5}x - 3$

Example 1

Find the equation of the line
with $m = 4$
and the y intercept is $(0,-5)$
$b = -5$

$y = mx + b$
$y = 4x - 5$

Example 2

Find the equation of the line
with $m = -3$
and the y intercept is $(0,6)$
$b = 6$

$y = mx + b$
$y = -3x + 6$

Example 3

Find the equation of the line
with $m = \frac{4}{5}$
and the y intercept is $(0,-1)$
$b = -1$

$y = mx + b$
$y = \frac{4}{5}x - 1$

Example 4

Find the equation of the line
with $m = \frac{5}{3}$
and the y intercept is $(0,0)$
$b = 0$

$y = mx + b$
$y = \frac{5}{3}x$

Finding the Equation of a Line
when the lines slope is a non zero number
and a point on the line that IS NOT the y intercept of the line
(finding the y intercept b)

If the **slope** of a line is a **non zero number** then the form of the equation will be $y = mx + b$ where m is the slope and $(0,b)$ is the y intercept. The x coordinate of the point **MUST** be zero for the point to be the y intercept. In the examples above the point given was always $(0,b)$ so the y intercept was b.

If you are given m as the slope of the line m and a point the line goes through (x_1,y_1) where **x IS NOT Zero** then you will need to find the value of the y intercept b.

Finding the value of b
given a non zero slope m and a point (x_1,y_1) that the line passes through.

Step 1. Write $y = mx + b$

Step 2. Substitute the known values of m, x_1 and y_1 into $y = mx + b$

Step 3. Solve for b

Finding the Equation Of A Line

Step 4: After finding b you will then substitute the values for m and b into the equation $y = mx + b$

Find the equation of the line given m (a non zero number) and a point (x_1,y_1) on the line.

Example 1

$m = 5$
and the line goes through
$(2,6)$

Step1. $y = mx + b$

Step 2. $6 = 5(2) + b$

Step 3. $6 = 10 + b$
$-4 = b$

Step 4. $y = 5x - 4$

Example 2

$m = -4$
and the line goes through
$(-2,-3)$

Step1. $y = mx + b$

Step 2. $-3 = -4(-2) + b$

Step 3. $-3 = 8 + b$
$-11 = b$

Step 4. $y = -4x - 11$

Find the equation of the line given m (a non zero number) and a point (x_1, y_1) on the line.

Step 1. Write $y = mx + b$

Step 2. Substitute the value of x_1 in for x and y_1 in for y and m for m into $y = mx + b$

Step 3. Solve for b

Step 4. Substitute m and b into $y = mx + b$

Example 3

$$m = \frac{2}{3}$$

and the line goes through

$$(9, -1)$$

Step1. $y = mx + b$

Step 2. $-1 = 9\left(\frac{2}{3}\right) + \text{b}$

Step 3. $-6 = 6 + \text{b}$

$-12 = b$

Step 4. $y = \frac{2}{3}x - 12$

Example 4

$$m = \frac{-3}{4}$$

and the line goes through

$$(-8, 1)$$

Step1. $y = mx + b$

Step 2. $1 = -8\left(\frac{-3}{4}\right) + \text{b}$

Step 3. $1 = 6 + \text{b}$

$-5 = b$

Step 4. $y = \frac{-3}{4}x - 5$

Finding the Equation of a Line given two points the line passes through.

The type of equation a line has is based on the slope of the line.

There are three different kinds of linear equations possible. Each linear equation corresponds to one of three different possible slopes. This means that **we must know the slope of the line** to know which one of the three types of equations to use for the equation. All of the examples above started with the slope of a line being known. If we are not given the slope of the line and a point the line passes through how can we find the equation of the line.

If you are given two points (x_1, y_1) and (x_2, y_2) that the line passes through find the slope of the line using the slope formula

Find m using $m = \frac{y_2 - y_1}{x_2 - x_1}$

Now you know the slope of the line m and one point the line passes through. In fact you know two points the line passes through. Use the value of the slope to determine the form of the equation of the line. Follow the example above to complete the equation of the line.

If you are given two points (x_1, y_1) and (x_2, y_2) the line passes through

Step 1: Find m using $m = \frac{y_2 - y_1}{x_2 - x_1}$

Step 2: Use the slope m of the line to determine which type of equation to use

Lines that have a zero slope	**Lines that have an undefined slope**	**Lines that have a slope m that is a non zero number**
have equations of the form	have equations of the form	have equations of the form
y = a constant	x = a constant	$y = mx \pm b$

Step 3: Use the slope you just found and **either one** of the two points (x_1, y_1) **or** (x_2, y_2) given and proceed to find the equation of the line just like you did in the previous problems.

Find the equation of the line given two points
(x_1, y_1) **and** (x_2, y_2)
the line passes through.

Start by finding the slope

Find m using $m = \frac{y_2 - y_1}{x_2 - x_1}$

Case 1: If m = 0

If the two points (x_1, y_1) and (x_2, y_2) have the same y coordinates $y_1 = y_2$

then the slope of the line will be 0 M = 0

and the equation of the line is

$$y = y_1$$

Note: You could also say that the equation of the line is $y = y_2$ but since $y_1 = y_2$ the answer will be the same either way.

Example 1

the line passes through
(5,6) and (2,6)

Find m $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{6-6}{2-5} = \frac{0}{-3} = 0$$

STOP
y = 6

Example 2

the line passes through
(2,–4) and (–2,–4)

Find m $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{-4+4}{-2-2} = \frac{0}{-4} = 0$$

STOP
$y = -4$

Find the equation of the line given two points

(x_1, y_1) **and** (x_2, y_2)

the line passes through.

Start by finding the slope

Find m using $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Case 2: If m is undefined

If the two points (x_1, y_1) and (x_2, y_2) have the same x coordinates $x_1 = x_2$

then the slope of the line m will be undefined

and the equation of the line is

$$x = x_1$$

Note: You could also say that the equation of the line is $x = x_2$ but since $x_1 = x_2$ the answer will be the same either way.

Example 1

the line passes through
(5,8) and (5,6)

Find m $m = \dfrac{y_2 - y_2}{x - x_1}$

$$m = \frac{6-8}{5-5} = \frac{-2}{0} = \text{undefined}$$

STOP

$$x = 5$$

Example 2

the line passes through
(–2,–4) and (–2,–9)

Find m $m = \dfrac{y_2 - y_2}{x - x_1}$

$$m = \frac{-9+4}{-2+2} = \frac{-5}{0} = \text{undefined}$$

STOP

$$x = -2$$

Find the equation of the line given two points

(x_1,y_1) **and** (x_2,y_2)

the line passes through.

Start by finding the slope

Find m using $m=\dfrac{y_2-y_1}{x_2-x_1}$

Case 3: **If m is a non zero number** then you must use the $y=mx+b$ form.

Step 1. Write $y=mx+b$

Step 2. Substitute the values of x_1 in for x and y_1 in for y and m for m into $y=mx+b$

Step 3. Solve for b

Step 4. Substitute m and b into $y=mx+b$

Example 1

Find the equation of the line that passes through $(4,8)$ and $(6,2)$

Find m $m=\dfrac{y_2-y_2}{x_2-x_1}$

$m=\dfrac{2-8}{6-4}=\dfrac{-6}{2}=-3$

Step1. $y=mx+b$

Step 2. $8=-3(4)+b$

Step 3. $8=-12+b$

$20=b$

Step 4. $y=-3x+20$

Example 2

Find the equation of the line that passes through $(-4,2)$ and $(-2,1)$

Find m $m=\dfrac{y_2-y_2}{x_2-x_1}$

$m=\dfrac{1-2}{-2+4}=\dfrac{-1}{2}$

Step1. $y=mx+b$

Step 2. $2=-4\left(\dfrac{-1}{2}\right)+b$

Step 3. $2=2+b$

$0=b$

Step 4. $y=\dfrac{-1}{2}x$

Problems involving Parallel or Perpendicular Lines

Example 1

Find the equation of the line that passes through $(4,7)$ and is parallel to $y = 2x - 5$

the slope of $y = 2x - 5$ is $m = 2$

a line parallel to $y = 2x - 5$ also has a slope of 2

Our line has a slope of 2 and goes through $(4,7)$

Step1. $y = mx + b$

Step 2. $7 = 2(4) + b$

Step 3. $7 = 8 + b$

$-1 = b$

Step 4. $y = 2x - 1$

Example 2

Find the equation of the line that passes through $(4,7)$ and is perpendicular to $y = 2x - 5$

the slope of $y = 2x - 5$ is $m = 2$

the slope of the perpendicular line is $\frac{-1}{2}$

Our line has a slope of $\frac{-1}{2}$ and goes through $(4,7)$

Step1. $y = mx + b$

Step 2. $7 = \frac{-1}{2}(4) + b$

Step 3. $7 = -2 + b$

$9 = b$

Step 4. $y = \frac{-1}{2}x + 9$

Example 3

the line passes through (5,2) and is parallel to $y = 3$

a line parallel to $y = 3$ will also have the form y = a number

$y = 2$

Example 4

the line passes through (5,2) and is perpendicular to $y = 7$

a line perpendicular to $y = 7$ will have the form x = a number

$x = 5$

