

## Section 4 – 3: Slope Introduction

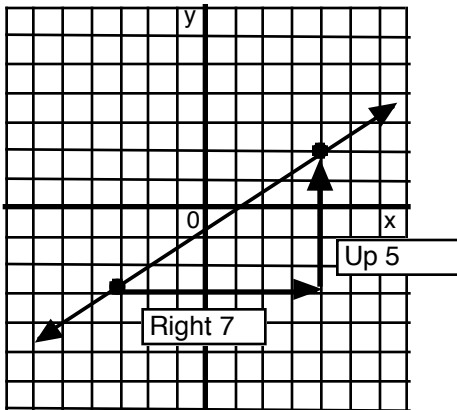
We use the term **Slope** to describe how steep a line is as you move between any two points on the line. The **slope** or steepness is a **ratio** of the vertical change in y (rise) compared to the horizontal change in x (run) between any two points on the line.

The letter m is used to denote the slope and we say that  $m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in y}}{\text{change in x}}$

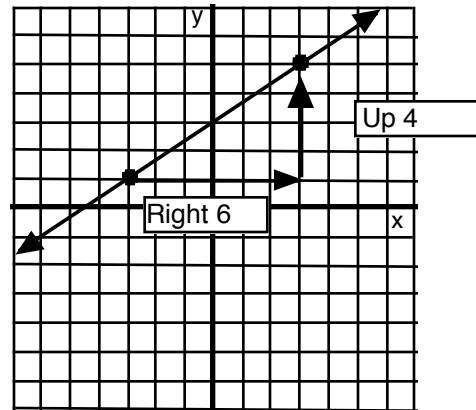
### Positive Slopes

**Lines that move up** as you move **from a point on the left to a point on the right** will have a **positive slope**. If you move to the right in the x direction the run will be a positive number. If you then move up in the y direction the rise will be a positive number. The ratio of a positive number divided by a positive number will make the slope a positive number.

$$1. \quad m = \frac{\text{change in y}}{\text{change in x}} = \frac{5}{7}$$



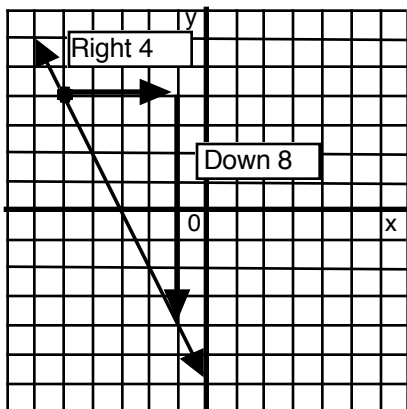
$$2. \quad m = \frac{\text{change in y}}{\text{change in x}} = \frac{4}{6} = \frac{2}{3}$$



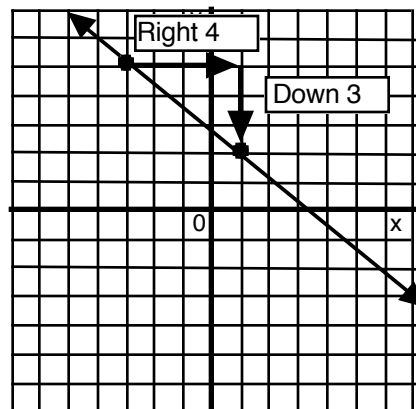
### Negative Slopes

**Lines that move down** as you move **from a point on the left to a point on the right** will have a **negative slope**. If you move to the right in the x direction the run will be a positive number. If you then move down in the y direction the rise will be a negative number. The ratio of a negative number divided by a positive number will make the slope a negative number.

$$3. \quad m = \frac{\text{change in y}}{\text{change in x}} = \frac{-8}{4} = \frac{-2}{1}$$



$$4. \quad m = \frac{\text{change in y}}{\text{change in x}} = \frac{-3}{4}$$

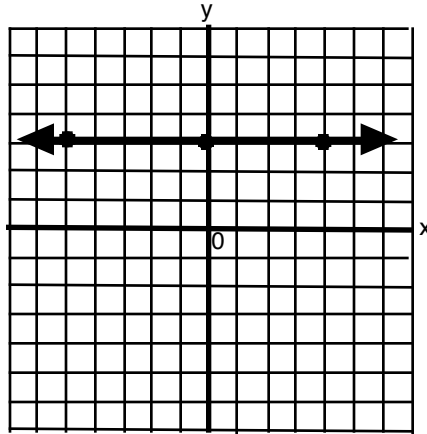


### Slopes of horizontal lines ( $m = 0$ )

If the points are all on a horizontal line then there will be a zero rise between the points.  
The slope will have a zero change in y

There will be a **zero on the top of the fraction** and **m will equal 0**

$$m = \frac{\text{change in y}}{\text{change in x}} = \frac{0}{\text{a nonzero number}} = 0 \text{ slope}$$

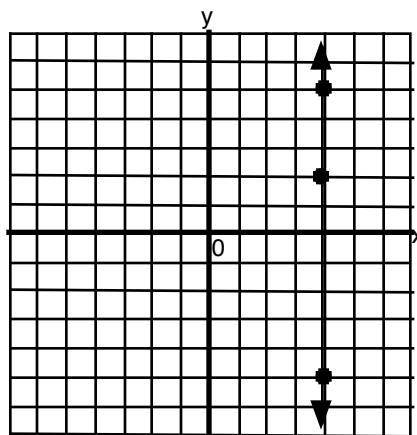


### Slopes of vertical lines ( $m$ is undefined)

If the points are all on a vertical line then there will a zero change in the run.  
The slope will have a zero change in x

There will be a **zero on the bottom of the fraction** and **m will be undefined**

$$m = \frac{\text{change in y}}{\text{change in x}} = \frac{\text{a nonzero number}}{0} = \text{undefined slope}$$

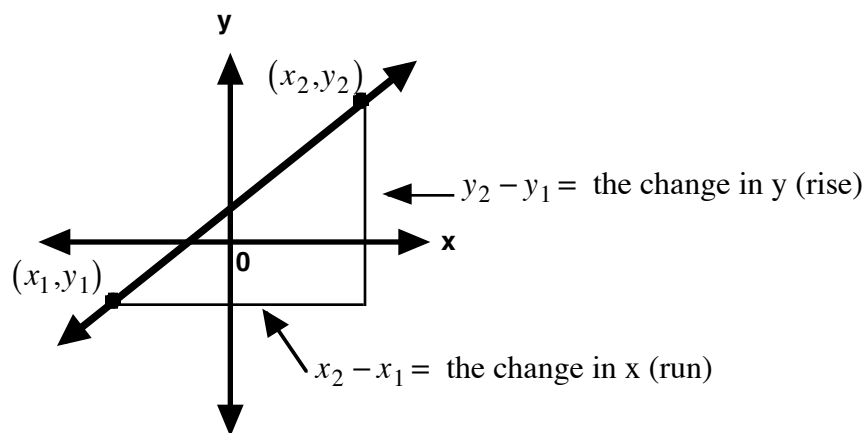


## The Slope Formula

We use the term **Slope** to describe how steep a line is as you move between any two points on the line. The **slope** or steepness is a **ratio** of the **vertical change (rise)** in  $y$  between these 2 points divided by the **horizontal change (run)** in  $x$  between these 2 points. It is common to start with the left most point on the line and move towards the right most point on the line to determine the **vertical change** in  $y$  and the **horizontal change (run)** in  $x$ .

The slope is a ratio (fraction) with the **numerator describing the vertical distance** between 2 points and the **denominator describing the horizontal change** between the same 2 points.

The two points will be shown as  $(x_1, y_1)$  and  $(x_2, y_2)$  where  $x_1$  and  $y_1$  are the coordinates of the first point and  $x_2$  and  $y_2$  are the coordinates of the second point.



The **vertical change** in  $y$  can be found by subtracting the  $y$  coordinates  $y_2 - y_1$  in that order.

The **horizontal change** in  $x$  can be found by subtracting the  $x$  coordinates  $x_2 - x_1$  in that order.

The ratio of the vertical change (rise) in the line compared to the horizontal change (run) is expressed as the slope.

Traditionally **the letter  $m$**  is used to represent the slope.

## The Slope Formula

For any two points  $(x_1, y_1)$  and  $(x_2, y_2)$

$$\text{the slope } m = \frac{y_2 - (y_1)}{x_2 - (x_1)}$$

Remember:  $-(y_1)$  means **if  $y_1$  is negative then  $-(y_1)$  will be positive**

$-(x_1)$  means **if  $x_1$  is negative then  $-(x_1)$  will be positive**

## Slopes of horizontal lines ( $m = 0$ )

If two points are on a horizontal line then they will have the same y coordinates.

**The difference  $y_2 - (y_1)$  will be 0**

If there a **zero on the top** of the fraction **the slope  $m$  will equal 0**

## Slopes of vertical lines ( $m$ is undefined)

If two points are on a vertical line then they will have the same x coordinates

The difference  $x_2 - (x_1)$  will be 0

If there a **zero on the bottom** of the fraction **the slope  $m$  will be undefined**

### Finding the slope of a line given 2 points

1. Use the slope formula  $m = \frac{y_2 - (y_1)}{x_2 - (x_1)}$  to find the slope of the line with the given points.
2. Reduce each ratio but do not change an improper fraction to a mixed number.

#### Example 1

Find the slope of the line through

(1, 5) and (3, 8)

$(x_1, y_1)$  and  $(x_2, y_2)$

$$m = \frac{8 - 5}{3 - 1}$$

$$m = \frac{3}{2}$$

#### Example 2

Find the slope of the line through

(-4, -5) and (-2, 4)

$(x_1, y_1)$  and  $(x_2, y_2)$

$$m = \frac{4 - (-5)}{-2 - (-4)}$$

$$m = \frac{4 + 5}{-2 + 4} = \frac{9}{2}$$

#### Example 3

Find the slope of the line through

(4, 2) and (2, 10)

$(x_1, y_1)$  and  $(x_2, y_2)$

$$m = \frac{10 - 2}{2 - 4}$$

$$m = \frac{8}{-2} = \frac{-4}{1}$$

#### Example 4

Find the slope of the line through

(-1, 3) and (1, 9)

$(x_1, y_1)$  and  $(x_2, y_2)$

$$m = \frac{9 - 3}{1 - (-1)}$$

$$m = \frac{6}{-2} = \frac{-3}{1}$$

#### Example 5

Find the slope of the line through

(-5, -3) and (-2, -3)

$(x_1, y_1)$  and  $(x_2, y_2)$

$$m = \frac{-3 - (-3)}{-2 - (-5)}$$

$$m = \frac{-3 + 3}{-2 + 5} = \frac{0}{3} = 0$$

#### Example 6

Find the slope of the line through

(2, 1) and (2, 3)

$(x_1, y_1)$  and  $(x_2, y_2)$

$$m = \frac{3 - 1}{2 - 2}$$

$$m = \frac{2}{0} = \text{undefined}$$

## Finding the slope of a line given the equation of the line

Finding the slope and y-intercept for equations with both an x and y term that have already been solved for y.

The most useful form for the equation of a line is the equation has been solved for y. If the equation has been solved for y then y is alone on one side of the equation and the other side will have an x term and a constant term. The **coefficient in front of the x term is the slope** of the line. We use the letter m for the slope. The constant term is the y coordinate of the y-intercept. The y-intercept is the **point** where the graph of the line crosses (or intercepts) the y axis. We use the letter b for the constant term.

### The Slope – y-Intercept form of the equation of a line

$$y = mx + b$$

When the equation of a line has been solved for y then  
**the number in front of the x term is called m (the slope)**  
**and the constant is the y coordinate of the y-intercept and is called b.**

The **slope of the line is m** and the **y-intercept is the point (0, b)**

To find the y slope and and y-intercept of a linear equation with x and y terms:

1. Get the equation in  $y = mx + b$  form by solving for y.
2. The slope will be m (the number in front of the x term)
3. The y-intercept will be the point **(0, b)**

#### Example 1

$$y = \frac{3}{4}x + 2$$

$$m = \frac{3}{4}$$

y - intercept is (0,2)

#### Example 2

$$y = -3x - 4$$

$$m = -3 \text{ or } \frac{-3}{1}$$

y - intercept is (0,-4)

#### Example 3

$$y = -x - 4$$

$$m = -1 \text{ or } \frac{-1}{1}$$

y - intercept is (0,-4)

#### Example 4

$$y = \frac{1}{2}x - 5$$

$$m = \frac{1}{2}$$

y - intercept is (0,-5)

#### Example 5

$$y = \frac{-4}{5}x$$

$$m = \frac{-4}{5}$$

y - intercept is (0,0)

#### Example 6

$$y = x$$

$$m = 1 \text{ or } \frac{1}{1}$$

y - intercept is (0,0)

## Finding the slope for equations with an x and y term that have not been solved for y.

Equations where **both an x and y terms** have not been solved for y require that you solve for y (get y all alone) first. After the equation has been solved for y then the coefficient (number in front of the x term) is the slope of the line.

### Example 1

Find the slope and the y - intercept

$$3x + 4y = 8 \quad (\text{Solve for } y)$$

$$\begin{array}{r} 3x + 4y = 8 \quad (\text{subtract } 3x \text{ from both sides}) \\ -3x \qquad -3x \end{array}$$

$$4y = -3x + 8 \quad (\text{divide by } 4 \text{ on both sides})$$

$$\frac{4y}{4} = \frac{-3}{4}x + \frac{8}{4}$$

$$y = \frac{-3}{4}x + 2$$

$$m = \frac{-3}{4}$$

the y - intercept is (0,2)

### Example 2

Find the slope and the y - intercept

$$-5x + 2y = -10 \quad (\text{Solve for } y)$$

$$\begin{array}{r} -5x + 2y = -10 \quad (\text{add } 5x \text{ to both sides}) \\ +5x \qquad +5x \end{array}$$

$$2y = 5x - 10 \quad (\text{divide by } 2 \text{ on both sides})$$

$$\frac{2y}{2} = \frac{5}{2}x - \frac{10}{2}$$

$$y = \frac{5}{2}x - 5$$

$$m = \frac{5}{2}$$

the y - intercept is (0,-5)

### Example 3

Find the slope and the y - intercept

$$2x - 5y = -15 \quad (\text{Solve for } y)$$

$$\begin{array}{r} 2x - 5y = -15 \quad (\text{subtract } 2x \text{ from both sides}) \\ -2x \qquad -2x \end{array}$$

$$-5y = -2x - 15 \quad (\text{divide by } -5 \text{ on both sides})$$

$$\frac{-5y}{-5} = \frac{-2}{-5}x - \frac{15}{-5}$$

$$y = \frac{2}{5}x + 3$$

$$m = \frac{2}{5}$$

the y - intercept is (0,3)

### Example 4

Find the slope and the y - intercept

$$-4x - 3y = 12 \quad (\text{Solve for } y)$$

$$\begin{array}{r} -4x - 3y = 12 \quad (\text{add } 4x \text{ to both sides}) \\ +4x \qquad +4x \end{array}$$

$$-3y = 4x + 12 \quad (\text{divide by } -3 \text{ on both sides})$$

$$\frac{-3y}{-3} = \frac{4}{-3}x + \frac{12}{-3}$$

$$y = \frac{-4}{3}x - 4$$

$$m = \frac{-4}{3}$$

y - intercept is (0,-4)

## Equations of the form $y = \text{constant}$

Equations with an  $y$  term but no  $x$  term represent **horizontal lines** and have **a slope of 0**. These lines are horizontal. They cross the  $y$  axis but not the  $x$  axis.

### Example 1

$$y = -3$$

and

$$m = 0$$

### Example 2

$$y = 6$$

and

$$m = 0$$

### Example 3

$$y - 2 = 4$$

converts to  $y = 6$

and

$$m = 0$$

## Equations of the form $x = \text{a constant}$

Equations with an  $x$  term but no  $y$  term represent **vertical lines** and have **a slope that is undefined**. These lines are vertical. They cross the  $x$  axis but not the  $y$  axis.

### Example 4

$$x = 9$$

and

$$m \text{ is undefined}$$

### Example 5

$$x = -3$$

and

$$m \text{ is undefined}$$

### Example 6

$$x + 9 = 0$$

converts to  $x = -9$

and

$$m \text{ is undefined}$$

## Parallel Lines

### Two Parallel Lines Have The Same Slope

**Two Parallel Lines** have the exact **same slope**. If you know the slope of one of the lines then the other line has the exact same slope.

#### Example 7

Line h has a slope of  $\frac{3}{5}$ . If Line k is parallel to line h then line k has a slope of  $\frac{3}{5}$  also.

#### Example 8

Line h has a slope of  $\frac{-2}{7}$ . If Line k is parallel to line h then line k has a slope of  $\frac{-2}{7}$  also.

#### Example 9

Line h has a slope of 4. If Line k is parallel to line h then line k has a slope of 4 also.

#### Example 10

Line h has a slope of -1. If Line k is parallel to line h then line k has a slope of -1 also.

## Perpendicular Lines

Perpendicular lines have slopes that are opposite in sign and whose numbers are inverses (negative reciprocals) of each other

Examples of negative reciprocals:

**Example 1**

$$\frac{2}{3} \text{ and } \frac{-3}{2}$$

**Example 2**

$$-2 \text{ and } \frac{1}{2}$$

**Example 3**

$$\frac{-1}{3} \text{ and } 3$$

**Example 4**

$$\frac{-3}{5} \text{ and } \frac{5}{3}$$

**Perpendicular Lines** have slopes that are opposite in sign and whose numbers are inverses of each other.

If you are given the slope of one of the lines then to find the other lines slope you **change the sign** and **invert (flip) the number** given for the first slope.

**Example 5**

Line h has a slope of  $\frac{3}{5}$  If Line k is perpendicular to line h then line k has a slope of  $\frac{-5}{3}$

**Example 6**

Line h has a slope of  $\frac{-2}{7}$  If Line k is perpendicular to line h then line k has a slope of  $\frac{7}{2}$

**Example 7**

Line h has a slope of  $\frac{1}{4}$  If Line k is perpendicular to line h then line k has a slope of  $-4$

**Example 8**

Line h has a slope of  $5$  If Line k is perpendicular to line h then line k has a slope of  $\frac{-1}{5}$