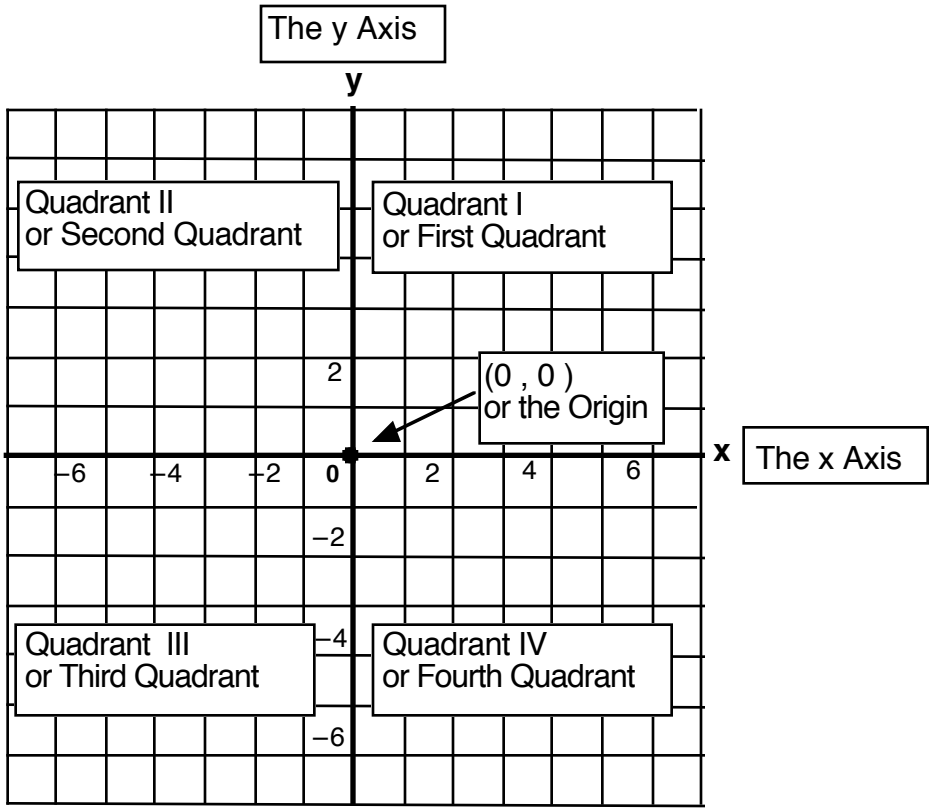
Section 4 – 1: The Rectangular Coordinate System

The **rectangular coordinate system** is based on two number lines. A **horizontal number line** called the **x axis** and a **vertical number line** called the **y axis**. Each axis has marks on them that represent a distance from the zero point on each axis. It is common for each mark to stand for one unit of distance. If a vertical or horizontal line is extended through each mark on each axis a grid is formed. The x and y axis are made darker so it is clear where each axis is. The two axis meet at the place where each axis has a zero (0) on its number line. This point is called **the origin.** Together the x and y axis break the coordinate grid into 4 areas called **quadrants.** The quadrants are numbered counterclockwise as shown. Points that are on either the x axis or the y axis are not considered to be in any of the quadrants but are said to be on an axis. Points not on the x axis or the y axis are in one of the four quadrants.

The Coordinate System



The Coordinate System

It is very important to remember that the picture above does not end in any direction. Since both the x axis and the y axis are really number lines they extend in both directions with no end. We often only show a small part of the entire **Coordinate System** for any problem.

The coordinate system is often called the **Cartesian Coordinate System** in honor of **Rene Descartes** (1596 – 1650). Be sure to have your instructor relate some of the stories about the life of this famous mathematician.

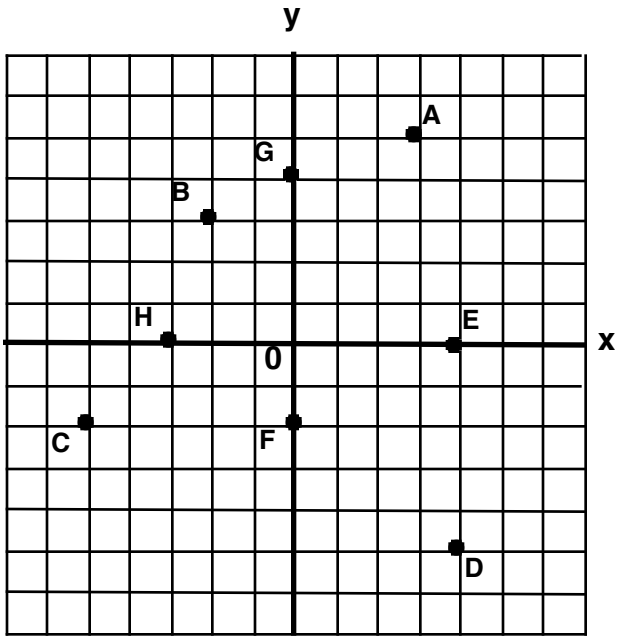
Graphing Ordered Pairs Introduction

Ordered Pairs: A pair of numbers written in the form (2, 3) is called an **ordered pair.** The first number in the parentheses is the value of the x coordinate and the second number is the value of the y coordinate. An ordered pair gives the location of a point on the coordinate grid.

Plotting Ordered Pairs on the Coordinate Grid

Each ordered pair represents a point on the coordinate grid. We use a single capital letter next to a dot to show the position of the point on the graph. To graph a point you start at (0, 0), the origin, and then move right or left based on the first number in the ordered pair. You move right if the first number is positive and left if the first number is negative. You then move up or down from there based on the second number in the ordered pair. You move up if the second number is positive and down if the second number is negative.

1. Graph **point A** at (3, 5) Start at (0, 0) and move **right 3** and **up 5** to plot point A
2. Graph **point B** at (–2, 3) Start at (0, 0) and move **left 2** and **up 3** to plot point B
3. Graph **point C** at (–5, –2) Start at (0, 0) and move **left 5** and **down 2** to plot point C
4. Graph **point D** at (4, –5) Start at (0, 0) and move **right 4** and **down 5** to plot point D
5. Graph **point E** at (4, 0) Start at (0, 0) and move **right 4** and **up 0** to plot point E
6. Graph **point F** at (0, –2) Start at (0, 0) and move **right 0** and **down 2** to plot point F
7. Graph **point G** at (0, 4) Start at (0, 0) and move **right 0** and **up 4** to plot point G.
8. Graph **point H** at (–3, 0) Start at (0, 0) and move **left 3** and **up 0** to plot point H



Point A is in the **first quadrant**. Point B is in the **second quadrant.** Point C is in the **third quadrant**. Point D is in the **fourth quadrant.** Point E and point H are on the **x axis**. Point G and Point F are on the **y axis**.

Solutions To LInear Equations in Two Variables

Linear equations have an x and y term that are both first degree terms (exponent of 1). An ordered pair is a **solution to a linear equation** if when you put the given number values for x and y into the equation in place of the variables the equation is true. Every linear equation has an infinite (unlimited) number of ordered pairs of x and y values that are solutions to the equation. We will test a given ordered pair to see if it is **one of the many solutions** for that equation.

Example 1

Is (4, 2) a solution to $y = 2x - 6$

(4, 2) means **x = 4** and **y = 2**

putting these values into $y = 2x - 6$ yields

$2 = 2(4) - 6$ which is **true**

so (4, 2) is a solution to $y = 2x - 6$

Example 2

Is (2, 5) a solution to $y = -3x + 2$

(2, 5) means **x = 2** and **y = 5**

putting these values into $y = -3x + 2$ yields

$5 = -3(2) + 2$ which is **false**

so (2, 5) is not a solution to $y = -3x + 2$

Example 3

Is (–6, 9) a solution to $y = \frac{2}{3}x + 1$

(–6, 9) means **x = –6** and **y = 9**

putting these values into $y = \frac{2}{3}x + 1$ yields

$9 = \frac{2}{3} \bullet \frac{-6}{1} + 1$ which reduces to

$9 = -4 + 1$ which is **false**

so (–6, 9) is not solution to $y = \frac{2}{3}x + 1$

Example 4

Is $\left(\frac{2}{3},\ 3\right)$ a solution to $y = -6x + 7$

$\left(\frac{2}{3},\ 3\right)$ means $x = \frac{2}{3}$ and y = 3

putting these values into $y = -6x + 7$ yields

$3 = \frac{-6}{1} \bullet \frac{2}{3}x + 7$ which reduces to

$3 = -4 + 7$ which is **true**

so $\left(\frac{2}{3},\ 3\right)$ **is a solution to** $y = -6x + 7$

Completing Ordered Pairs

The last section gave you a linear equation and an ordered pair of x and y coordinates and asked you to determine if the given ordered pairs were solutions to the equation. There are an infinite (unlimited) number of ordered pairs that are solutions for any given linear equation. For any given x coordinate there is a y coordinate that will make the equation true. Likewise, for any given y coordinate there is an x coordinate that will make the equation true. In this section we will list a linear equation and give you an x coordinate and you will need to find the missing y coordinate.

Finding y given x

If you are given a linear equation in the form $y = mx + b$ and the value for y is given then you find the value for x by replacing the given y value into the equation in place of y and solve for x. List the completed ordered pair in the correct order inside a pair of parenthesis. You can check your answer by putting both the x and y coordinates into the equation and checking to see if it results in a true equation.

Example 1: Finding y given x

Complete each of the given ordered pairs for the equation

$y = 2x - 3$ for (4, ____) and (–5, _____)

(4 , __) means find y if x $= 4$

If $y = 2x - 3$ and x $= 4$
then replace x with 4

$y = 2(4) - 3$
$y = 8 - 3$
$y = 5$
(4,5) is a solution to
$y = 2x - 3$

(-5 , __) means find y if x $= -5$

If $y = 2x - 3$ and x $= -5$
then replace x with -5

$y = 2(-5) - 3$
$y = -10 - 3$
$y = -13$
(–5,–13) is a solution to
$y = 2x - 3$

Example 2 : Finding y given x

Complete each of the given ordered pairs for the equation

$$y = \frac{-2}{3}x + 5 \text{ for } (6, \_\_\_) \text{ and } (-3, \_\_\_)$$

(6 , __) means find y if x $= 6$

If $y = \frac{-2}{3}x + 5$ and x $= 6$

then replace x with 6

$$y = \frac{-2}{3}\left(\frac{6}{1}\right) + 5$$

$$y = -4 + 5$$

$$y = 1$$

(6,1) is a solution to

$$y = \frac{-2}{3}x + 5$$

(-3 , __) means find y if x $= -3$

If $y = \frac{-2}{3}x + 5$ and x $= -3$

then replace x with -3

$$y = \frac{-2}{3}\left(\frac{-3}{1}\right) + 5$$

$$y = 2 + 5$$

$$y = 7$$

(−3,7) is a solution to

$$y = \frac{-2}{3}x + 5$$

Completing Ordered Pairs

Finding x given y

If you are given a linear equation in the form $y = mx + b$ and value of the y is given then find the value for x by replacing the given y value in the equation and solve for x. List the completed ordered pair in the correct order inside a pair of parenthesis. You can check your answer by putting both the x and y coordinates into the equation and checking to see if it results in a true equation.

Example 1: Finding x given y

Complete each of the given ordered pairs for the equation

$y = 2x + 3$ for (__ , 7) and (__ , –5)

(__ , 7) means find x if y = 7

If $y = 2x + 3$ and y = 7
then replace y with 7

$7 = 2x + 3$
and solve for x by
subtracting 3 from both sides

$4 = 2x$
(divide both sides by 2)

$2 = x$
(2, 7) is a solution to
$y = 2x + 3$

(__ , −5) means find x if y = –5

If $y = 2x + 3$ and y = −5
then replace y with −5

$-5 = 2x + 3$
and solve for x by
subtracting 3 from both sides

$-8 = 2x$
(divide both sides by 2)

$-4 = x$
(–4,–5) is a solution to
$y = 2x + 3$

Example 2: Finding x given y

Complete each of the given ordered pairs for the equation

$$y = \frac{2}{3}x + 4 \text{ for } (\_\_, 6) \text{ and } (\_\_, -2)$$

(__ , 6) means find x if y = 6

If $y = \frac{2}{3}x + 4$ and y = 6

then replace y with 6

$$6 = \frac{2}{3}x + 4$$

and solve for x by multipling each term on both sides by 3 to eliminate fractions

$$(3)6 = (3)\frac{2}{3}x + (3)4$$

$$18 = 2x + 12$$

(subtract 12 from both sides)

$$6 = 2x$$

(divide both sides by 2)

$$3 = x$$

(3, 6) is a solution to $y = \frac{2}{3}x + 4$

(__ , −2) means find x if y = −2

If $y = \frac{2}{3}x + 4$ and y = −2

then replace y with −2

$$-2 = \frac{2}{3}x + 4$$

and solve for x by multipling each term on both sides by 3 to eliminate fractions

$$(3)(-2) = (3)\frac{2}{3}x + (3)4$$

$$-6 = 2x + 12$$

(subtract 12 from both sides)

$$-18 = 2x$$

(divide both sides by 2)

$$-9 = x$$

(−9, −2) is a solution to $y = \frac{2}{3}x + 4$

Completing Ordered Pairs

Finding x or y in Ax + By = C form

If you are given a linear equation in the form $Ax + By = C$ and value of the x is given then find the value for y by replacing the given x value in the equation and solve for y.

If you are given a linear equation in the form $Ax + By = C$ and value of the y is given then find the value for x by replacing the given y value in the equation and solve for x.

List the completed ordered pair as the final solution. You can check your answer by putting both the x and y coordinates into the equation and checking to see if it results in a true equation.

Example 1: Finding x given y.

Complete each of the given ordered pairs for the equation

$2x - 3y = 12$ for (____, 2) and (___ , 4)

(___ , 2) means find x if $y = 2$

If $2x - 3y = 12$ and $y = 2$
then replace y with 2

$2x - 3(2) = 12$
$2x - 6 = 12$
and solve for x

$2x - 6 = 12$
(add 6 to both sides)

$2x = 18$
(divide both sides by 2)

$x = 9$
(9,2) is a solution to
$2x - 3y = 12$

(___ , 4) means find x if $y = 4$

If $2x - 3y = 12$ and $y = 4$
then replace y with 4

$2x - 3(4) = 12$
$2x - 12 = 12$
and solve for x

$2x - 12 = 12$
(add 12 to both sides)

$2x = 24$
(divide both sides by 2)

$y = 12$
(12,4) is a solution to
$2x - 3y = 12$

Example 2: Finding y given x.

Complete each of the given ordered pairs for the equation

$-x-4y=6$ for $(-10, \_\_)$ and $(2, \_\_\_\_)$

$(-10, \_\_)$ means find y if $x=-10$

If $-x-4y=6$ and $x=-10$
then replace x with -10

$-(-10)-4y=6$
$10-4y=6$
and solve for y

$10-4y=6$
(subtract 10 from both sides)

$-4y=-4$
(divide both sides by -4)

$y=1$
$(-10,1)$ is a solution to
$-x-4y=6$

$(2, \_\_)$ means find y if $x=2$

If $-x-4y=6$ and $x=2$
then replace x with 2

$-(2)-4y=6$
$-2-4y=6$
and solve for y

$-2-4y=6$
(subtract 10 from both sides)

$-4y=8$
(divide both sides by -4)

$y=-2$
$(2,-2)$ is a solution to
$-x-4y=6$

Completing Tables Of Ordered Pairs

Ordered pairs are often displayed in a table format. The values for x are listed in the first column and the value for y is listed in the second column. If you know one of the values for x or y you can find the other value by replacing the known value into the equation and solving for the unknown.

Example 1:

Complete the table of ordered pairs shown below for the equation $y = 5x - 6$

X	Y
0	
1	
2	

If x = 0
and $y = 5x - 6$
then $y = 5(0) - 6$
and $y = 0 - 6$
and **y = –6**

If x = 1
and $y = 5x - 6$
then $y = 5(1) - 6$
and $y = 5 - 6$
and **y = –1**

If x = 2
and $y = 5x - 6$
then $y = 5(2) - 6$
and $y = 10 - 6$
and **y = 4**

X	Y
0	–6
1	–1
2	4

The completed table is shown at the left and shows the following ordered pairs **(0, –6) (1, –1) (2, 4)**

Example 2: Complete the table of ordered pairs shown below for the equation $y = \frac{-2}{3}x + 4$

X	Y
0	
3	
6	

If x = 0
and $y = \frac{-2}{3}x + 4$
then $y = \frac{-2}{3}(0) + 4$
and $y = 0 + 4$
and **y = 4**

If x = 3
and $y = \frac{-2}{3}x + 4$
is $y = \frac{-2}{3}(3) + 4$
and $y = -2 + 4$
and **y = 2**

If x = 6
and $y = \frac{-2}{3}x + 4$
is $y = \frac{-2}{3}(6) + 4$
and $y = -4 + 4$
and **y = 0**

X	Y
0	4
3	2
6	0

The completed table is shown at the left and shows the following ordered pairs **(0, 4) (3, 2) (6, 0)**

