

Section 3 – 3 B: Advanced Solving Word Problems

Coin and Tickets Problems

Coin and ticket problems involve finding the number coins or tickets of different values when given the total value or cost for all the coins or tickets. We will start to develop an equation we will use to solve these problems by looking at a similar type of example.

Note: we list money values as a decimal number with two decimal places.

Nickels = .05, Dimes = .10 and quarters = .25

If Mary had 12 dimes and 20 nickels an expression for the total value of the coins could be found by

1. the product of the number of dimes (12) by the value for 1 dime (.10) or $12(.10) = 1.20$
and adding
2. the product of the number of nickels (20) times the value for 1 nickel (.05) or $20 (.05) = 1.00$

adding these two products together gives us the total values of the coins

the final equation would look like this

$$12(.10) + 20 (.05) = \$ 2.20$$

In coin and ticket problems you will not be given the number of coins and asked to find the total value of the coins. Instead, you will be given the total value of all the coins together (the \$ 2.20 above) and asked to find the number of each different coin that will give you that total value.

The equation you will use is the same one you used above

$$\left(\begin{array}{l} \text{number of 1} \\ \text{type of coin} \end{array} \right) \left(\begin{array}{l} \text{value of 1 coin of} \\ \text{that type} \end{array} \right) + \left(\begin{array}{l} \text{number of the other} \\ \text{type of coin} \end{array} \right) \left(\begin{array}{l} \text{value of 1 coin of} \\ \text{that type} \end{array} \right) = \text{total of all coins}$$

You will be given the different **types** of coins (ie. nickels, dimes and/or quarters) and the total value of all the coins. You will be given an **expression** for the number of 1 type of coin in terms of the other type of coin (ie. There are **3 more dimes than nickels**) You will use that sentence to fill in an algebraic expression in terms of x for the number of coins in each place in the equation where the number of coins is listed and then solve for x.

Note : The value of money is shown as a decimal number with two decimal places. After we get the algebraic equation we can multiply each term by 100 (**move the decimal point 2 places to the right**) so that we can produce an equivalent equation without decimals.

Coins Problem Examples

Coins Problem Example 1:

Tom has 3 more dimes than he has nickels. He has \$ 1.65 total. Find how many of each type of coin he has.

Let the number of nickels = x

Let the number of dimes = $x + 3$ (3 more dimes than nickels)

$$\left(\begin{array}{l} \text{number of} \\ \text{type of coin} \end{array} \right) \left(\begin{array}{l} \text{value of 1 coin of} \\ \text{that type} \end{array} \right) + \left(\begin{array}{l} \text{number of the other} \\ \text{type of coin} \end{array} \right) \left(\begin{array}{l} \text{value of 1 coin of} \\ \text{that type} \end{array} \right) = \text{total of all coins}$$

nickels	dimes	
$x(.05)$	$+$	$(x + 3)(.10) = 1.65$

$x(5) + (x + 3)(10) = 165$	(multiply each term by 100 to eliminate decimals)
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$5x + 10x + 30 = 165$	distribute the 10
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$$5x = 135$$

$$x = 9 \text{ (the number of nickels)}$$

$$\text{and the number of dimes} = x + 3 = 9 + 3 = 12$$

answer: The number of nickels is 9 and the number of dimes is 12

$$\text{Check: } 9(.05) + 12(.10) = 1.65$$

Coins Problem Example 2:

Mary has 4 less than twice as many quarters than she has dimes. She has \$ 6.20 total. Find how many of each type of coin she has.

Let the number of dimes = x

Let the number of quarters = $2x - 4$ (4 less than twice as many quarters as dimes)

dimes		quarters		
$x (.10)$	+	$(2x - 4) (.25)$	=	6.20

$x(.10) + (2x - 4)(.25) = 6.20$ (multiply each term by 100 to eliminate decimals)

$x(10) + (2x - 4)(25) = 620$ distribute the 25

$10x + 50x - 100 = 620$

$60x - 100 = 620$

$60x = 720$

$x = 12$ (the number of dimes)

and the number of quarters = $2x - 4 = 2(12) - 4 = 20$

Answer: The number of dimes is 12 and the number of quarters is 20

Check: $12(.10) + 20(.25) = 1.20 + 5.00 = \$ 6.200$

Ticket/ Sales Problem Examples

Tickets/ Sales Problems are very similar to the coin problems above. There are 2 (or more) tickets or items to sell and each item is a different price. You will be given the price of each of the different items and the total dollar amount of the sells (the total receipts) and be asked to find the number of each item sold.

The equation you will use is shown below

$$\left(\begin{array}{l} \text{number of items} \\ \text{of 1 cost} \end{array} \right) \left(\begin{array}{l} \text{value of 1 item of} \\ \text{that type} \end{array} \right) + \left(\begin{array}{l} \text{number of items} \\ \text{at a different cost} \end{array} \right) \left(\begin{array}{l} \text{value of 1 item of} \\ \text{that type} \end{array} \right) = \text{total cost}$$

Ticket/ Sales Problem Example 1:

The Folsom Little League sells soda pop tickets priced at 20 cents and hot dog tickets priced at 60 cents each. They plan to sell 6 more hot dogs tickets than soda pop tickets. If they plan to take in \$9.20 how many tickets of each type do they need to sell?

Let the number of soda pop tickets = x

Let the number of hot dog tickets = $x + 6$ (Note: 6 more hot dog tickets as soda pop tickets)

$$\left(\begin{array}{l} \text{number of items} \\ \text{of 1 cost} \end{array} \right) \left(\begin{array}{l} \text{value of 1 item of} \\ \text{that type} \end{array} \right) + \left(\begin{array}{l} \text{number of items} \\ \text{at a different cost} \end{array} \right) \left(\begin{array}{l} \text{value of 1 item of} \\ \text{that type} \end{array} \right) = \text{total cost}$$

$(x) \quad (.20) \quad + \quad (x + 6) \quad (.60) \quad = 9.20$

soda pop	hot dog
$x (.20)$	$+ (x + 6) (.60)$
$= 9.20$	

$$x (.20) + (x + 6) (.60) = 9.20 \quad (\text{multiply each term by 100 to eliminate decimals})$$

$$x(20) + (x + 6)(60) = 920 \quad (\text{distribute the 60})$$

$$20x + 60x + 360 = 920$$

$$80x = 560$$

$$\mathbf{x = 7 \text{ soda pop tickets}}$$

$$\text{and the number of hot dog tickets} = x + 6 = 7 + 6 = 13$$

Answer:

The number soda pop tickets is 7 and the number of hot dog tickets is 13

$$\text{Check: } 7(.20) + 13(.60) = 1.40 + 7.80 = \$ 9.20$$

Ticket/ Sales Problem Example 2:

Dimple Records sells used CD's for \$ 2.50 each and new Cd's for \$ 8.00 each. One day they sold 10 more new CD's than used cd's. If the took had \$ 206 in total sells, how many of each type did they sell?

Let the number of used CD's = x

Let the number of new CD's = $x + 8$ Note: 8 more new CD's than used cd's.

$$\left(\begin{array}{l} \text{number of items} \\ \text{of 1 cost} \end{array} \right) \left(\begin{array}{l} \text{value of 1 item of} \\ \text{that type} \end{array} \right) + \left(\begin{array}{l} \text{number of items} \\ \text{at a different cost} \end{array} \right) \left(\begin{array}{l} \text{value of 1 item of} \\ \text{that type} \end{array} \right) = \text{total cost}$$

$$(x) \quad (2.50) \quad + \quad (x + 10) \quad (8.00) \quad = 190.00$$

2.50 CD 8.00 CD

$$x (2.50) + (x + 8) (8.00) = 190.00$$

$$x (2.50) + x + 8)(8.00) = 190.00 \quad (\text{multiply each term by 100 to eliminate decimals})$$

$$x (250) + (x + 8)(800) = 19000 \quad (\text{distribute the 800})$$

$$250x + 800x + 6400 = 19000$$

$$1050x + 6400 = 19000$$

$$1050x = 12600$$

$x = 12$ used CD's

and the number of used CD's is $x + 8 = 12 + 8 = 20$

Answer: The number of used CD's is 12 and the number of new CD's is 20

$$\text{Check: } 12(.2.50) + 20(.8.00) = 30 + 160 = \$ 196$$

Investment Examples

Investment Problems are very similar to the Tickets/ Sales problems above. There are 2 (or more) investment accounts and each account pays a different rate of simple annual interest. You will be given the rate of simple annual interest for each of the different accounts. You will then need to find the amount to invest in each of the accounts so the investments will return the annual interest stated in the problem.

The equation you will use is similar to the one used for coins and tickets

$$\left(\begin{array}{l} \text{amount of \$} \\ \text{in one account} \end{array} \right) \left(\begin{array}{l} \% \text{ interest} \\ \text{for that account} \end{array} \right) + \left(\begin{array}{l} \text{amount of \$} \\ \text{in 2nd account} \end{array} \right) \left(\begin{array}{l} \% \text{ interest} \\ \text{for that account} \end{array} \right) = \left(\begin{array}{l} \text{total annual} \\ \text{interest earned} \end{array} \right)$$

Investment Example 1:

David wants to invest **some** of his savings in a bank **that pays 5%** interest and **\$400** of his money in a CD **that pays 8%**. How much money should he put in the 5% account if he wants to earn \$ 82 total annual interest?

Let the amount at 5% = x

Let the amount at 8% = 400

$$\left(\begin{array}{l} \text{amount of \$} \\ \text{in one account} \end{array} \right) \left(\begin{array}{l} \% \text{ interest} \\ \text{for that account} \end{array} \right) + \left(\begin{array}{l} \text{amount of \$} \\ \text{in 2nd account} \end{array} \right) \left(\begin{array}{l} \% \text{ interest} \\ \text{for that account} \end{array} \right) = \left(\begin{array}{l} \text{total annual} \\ \text{interest earned} \end{array} \right)$$

5% 8%
account account

$$x (.05) + (400) (.08) = 82.00 \quad (\text{some at 5\% and 400 at 8\%})$$

$$x (.05) + (400) (.08) = 82.00 \quad (\text{multiply each term by 100})$$

$$x (5) + (400)(8) = 8200$$

$$5x + 3200 = 8200$$

$$5x = 5000$$

$$\mathbf{x = 1000}$$

Answer: \$1000 at 5% and \$400 at 8%

$$\text{Check: } 1000 (.05) + 400 (.08) = 50 + 32 = 82$$

Investment Example 2:

Ann Marie wants to invest **some** of her \$ 4,000 savings in a CD that **pays 8%** interest and **twice as much** in a CD that **pays 10%**. She wants to make \$ 1400 interest in one year. How much money should she put in each account.

Let the amount at 8% = x

Let the amount at 10% = 2x (twice as much)

$$\left(\begin{array}{l} \text{amount of \$} \\ \text{in one account} \end{array} \right) \left(\begin{array}{l} \% \text{ interest} \\ \text{for that account} \end{array} \right) + \left(\begin{array}{l} \text{amount of \$} \\ \text{in 2nd account} \end{array} \right) \left(\begin{array}{l} \% \text{ interest} \\ \text{for that account} \end{array} \right) = \left(\begin{array}{l} \text{total annual} \\ \text{interest earned} \end{array} \right)$$

8% account **10% account**

$$x(.08) + (2x)(.10) = 1400.00 \quad \text{(some at 8% and twice as much at 10%)}$$

$$x(.08) + 2x(.10) = 1400.00 \quad \text{(multiply each term by 100)}$$

$$x(8) + 2x(10) = 140,000$$

$$28x = 140,000$$

$$x = 5,000$$

Answer: \$ 5,000 at 8% and \$ 10,000 at 10%

Investment Example 3:

David wants to invest some of his savings in a bank that pays 5% interest and the rest of his money in a CD that pays 8%. He has \$ 22,000 to invest and wants to make \$ 1400 interest in one year. How much money should he put in each account?

Total money to invest – money in one amount = money in the other amount

$$\left(\begin{array}{l} \text{amount of \$} \\ \text{in one account} \end{array} \right) \left(\begin{array}{l} \% \text{ interest} \\ \text{for that account} \end{array} \right) + \left(\begin{array}{l} \text{amount of \$} \\ \text{in 2nd account} \end{array} \right) \left(\begin{array}{l} \% \text{ interest} \\ \text{for that account} \end{array} \right) = \left(\begin{array}{l} \text{total annual} \\ \text{interest earned} \end{array} \right)$$

Let: amount at 5% = x amount at 8% = 22,000 – x

5% account 8% account

$$x(.05) + (22,000 - x)(.08) = 1400 \quad \text{multiply each term by 100 to eliminate the decimal}$$

$$x(5) + (22,000 - x)(8) = 140,000$$

$$5x + 176,000 - 8x = 140,000$$

$$-3x = -36,000$$

$$x = 12,000$$

Answer: x = 12,000 at %5 and the rest (\$10,000) at %8

$$\text{Check: } 12,000(.05) + 10,000(.08) = 600 + 800 = 1400$$