

Section 3 – 2:

Let Statements

Algebraic Expressions For Two Quantities

Almost all word problems are based on a comparison between two (or more) unknown quantities. In any word problem, one quantity is being compared to another quantity. For example, a problem may state that “the **number of dogs** is 5 more than the **number of cats**.” You do not know the number of dogs or cats so a variable expression is needed for each quantity. One of the quantities will be set equal to x and the other quantity will be an expression with x in it.

Which of the two quantities should be set equal to x ?

We let the quantity that is being used as the basis for comparison equal x . When there are only two quantities in a sentence, **the quantity mentioned second** in the sentence is almost always the quantity that the other quantity is being compared to. This means that the **quantity mentioned second is set equal to x** and the first quantity mentioned will be an expression with x in it.

Example 1

Use a variable and two algebraic expressions to describe the quantities in the sentence below.

The **number of dogs** is 5 more than the **number of cats**.

The quantity mentioned **second** is the number of **cats**. The number of cats is set equal to x .

Let the number of cats = x

The number of **dogs is 5 more than** the number of **cats (x)** so

The number of dogs = $x + 5$

We now have an expression for each of the quantities in the sentence.

Example 2

Use a variable and two algebraic expressions to describe the quantities in the sentence below.

Sally ran 6 miles less than the number of miles Bill ran.

The quantity mentioned **second** is the **miles Bill ran**. The miles Bill ran is set equal to x .

Let the number of miles Bill ran = x

The **number of miles Sally ran is 6 less than** the number of miles **Bill ran (x)**.

Let the number of miles Sally ran = $x - 6$

Example 3

Use a variable and two algebraic expressions to describe the quantities in the sentence.

The **number of cars** is 3 less than twice the **number of bikes**.

The quantity mentioned second is **the number of bikes**. That quantity is set equal to x .

Let the number of bikes = x

The number of cars is **3 less than twice the number of bikes (x)**. Twice x is $2x$.

Let the number of cars = $2x - 3$

Example 4

Use a variable and two algebraic expressions to describe the quantities in the sentence.

The **length of a rectangle** is 12 less than three times **the width** of the rectangle.

The quantity mentioned **second** is **the width** of the rectangle. That quantity is set equal to x .

Let the width = x

The length is **12 less than three times** the width. The width is x so 3 times the width is $3x$.

Let the length = $3x - 12$

Example 5

Use a variable and two algebraic expressions to describe the quantities in the sentence.

Angle B is 28 degrees more than four times **Angle A**.

The quantity mentioned **second** is the **Angle A**. That quantity is set equal to x .

Let Angle A = x

Angle B is **28 degrees more than four times Angle A**. Angle A is x so 4 times Angle A is $4x$.

Let Angle B = $4x + 28$

Algebraic Expressions with Three Unknown Quantities.

Sometimes there are three unknown quantities mentioned in a problem. These problems will have **two sentences** that compare unknowns. One sentence is a comparison between the first two quantities. We use the first comparison to set up two expressions. The quantity mentioned **second is set equal to x** and the first quantity will be related to x just as we have done with the previous problems. The next sentence will compare a **third quantity to one of the two previous quantities**. You will relate this third quantity to the one it is compared to. It may be related to the first or second quantity so read the sentence carefully.

Example 6

Use a variable and three expressions to describe the quantities in the sentences.

The **number of hours Jim worked** is 5 more than **the number of hours Sam worked**. The **number of hours David worked** is 8 less than **the number of hours Sam worked**.

The first sentence says:

The **number of hours Jim worked** is 5 more than **the number of hours Sam worked**.

The quantity mentioned **second** is the number of **hours Sam worked**. That quantity is set equal to x.

Let the number of hours Sam worked = x

The **number of hours Jim worked** is being compared to the **number of hours Sam worked (x)**.

The **number of hours Jim worked** is **5 more than the number of hours Sam worked (x)**.

Let number of hours Jim worked = x + 5

The second sentence says:

The **number of hours David worked** is 8 less than **the number of hours Sam worked**.

In the second sentence the **number of hours David worked** is being compared to the number of hours Sam worked **(x)**.

The **number of hours David worked** is **8 less than the number of hours Sam worked (x)**

Let number of hours David worked = x - 8

Example 7

Use a variable and three expressions to describe the quantities in the sentences.

The **second angle** of a triangle is 5 more than the **first angle**. The **third angle** is 20 more than the **first angle**.

The first sentence says:

The **second angle** of a triangle is 5 more than the **first angle**.

The quantity mentioned **second** is **the first angle**. That quantity is set equal to x .

Let the **first angle** = x

The **second angle** is being **compared** to the **first angle** (x).

The **second angle** of a triangle is **5 more than the first angle** (x)

Let the **second angle** = $x + 5$

The second sentence says:

The **third angle** is 20 more than the **first angle**.

In the second sentence the **third angle** is being compared to the **first angle** (x)

The **third angle** is **20 more than the first angle** (x).

Let the **third angle** = $x + 20$

Example 8

Use a variable and three expressions to describe the quantities in the sentences.

The number of **cars** is 12 more than three times the number of **bikes**. The **number of buses** is 30 less than the **number of cars**.

The first sentence says:

In the first sentence the quantity mentioned **second** is **the number of bikes**. The **number of bikes** is set equal to x .

Let the number of bikes = x

In the first sentence the **number of cars** is being **compared** to the **number of bikes** (x).

The **number of cars** is **12 more than three times the number of bikes** (x)

Let the number of cars = $3x + 12$

The second sentence says:

The **number of buses** is being compared to the **number of cars**,

Note: the buses are compared to CARS which are $(3x + 12)$ not Bikes (x)

The **number of buses** is **30 less than the number of CARS**. Cars are $(3x + 12)$

Let the number of buses = $3x + 12 - 30$