

Section 2 – 2: Solving Equations With Multiple Steps

Solving Equations with an x term on both sides of the Equation

Three step equations

Sometimes a variable term and a constant term appear **on both** sides of an equation. To get x alone will take **3 steps**.

Step 1: Eliminate the **smallest variable term** by adding or subtracting the smallest variable term to both sides of the equation. **You will now have a two step equation**

Step 2: **Eliminate the constant term** that is **on the side with the variable term** by adding or subtracting it to both sides of the equation.

Step 3: Eliminate the **coefficient with the variable** by multiplying or dividing both sides of the equation by the coefficient.

Example 1

$$4x + 6 = 2x + 14$$

Step 1 $-2x \quad -2x$

$$2x + 6 = 14$$

Step 2 $-6 \quad -6$

$$2x = 8$$

Step 3 $\frac{2x}{2} = \frac{8}{2}$

$$x = 4$$

Example 2

$$3x + 8 = 6x + 14$$

Step 1 $-3x \quad -3x$

$$8 = 3x + 14$$

Step 2 $-14 \quad -14$

$$-6 = 3x$$

Step 3 $\frac{-6}{3} = \frac{3x}{3}$

$$-2 = x$$

Example 3

$$x + 6 = -2x + 15$$

Step 1 $+2x \quad +2x$

$$3x + 6 = 15$$

Step 2 $-6 \quad -6$

$$3x = 9$$

Step 3 $\frac{3x}{3} = \frac{9}{3}$

$$x = 3$$

It is not required that you eliminate the smallest variable term first. You could choose to eliminate the largest variable term first. The process works the same but if you choose to eliminate the largest variable term first you will divide by a negative number in step 3. Negative signs are a source of errors so eliminating the smallest variable term first helps reduce errors and is the preferred method.

Example 4

← same problem →

Example 5

Solve $5x + 10 = 3x + 16$

Solve $5x + 10 = 3x + 16$

by **eliminating the 3x** on both sides

by **eliminating the 5x** on both sides

$$5x + 10 = 3x + 16$$

Step 1 $-3x \quad -3x$

$$2x + 10 = 16$$

Step 2 $-10 \quad -10$

$$2x = 6$$

Step 3 $\frac{2x}{2} = \frac{6}{2}$
 $x = 3$

$$5x + 10 = 3x + 16$$

Step 1 $-5x \quad -5x$

$$10 = -2x + 16$$

Step 2 $-16 \quad -16$

$$-6 = -2x$$

Step 3 $\frac{-6}{-2} = \frac{-2x}{-2}$
 $3 = x$

The solution is the same in both cases.

Does Every Equation Have Only One Solution?

It would seem that every equation has exactly one value of x as a solution. Actually, there are two cases when an equation does not have exactly one solution. In one case there are **no numbers** that can be substituted in for x to make the equation true. In a second case any number you chose can be substituted in for x and it will be a solution for the equation so **all numbers** are a solution.

Equations With No Numbers As A Solution

When you are getting the x terms on the same side of an equation and the **x terms cancel out** leaving you a statement with only numbers and **that statement is FALSE** then the answer is
No Solution or \emptyset

Example 1

Solve For x : $4x + 6 = 4x + 14$

$$\begin{array}{r} 4x + 6 = 4x + 14 \text{ (subtract } 4x \text{ from} \\ -4x \quad -4x \quad \text{both sides)} \end{array}$$

$$6 = 14$$

the x terms have dropped out
and the the remaining statement
is FALSE so there are no numbers
that work and the answer is
No Solution

Example 2

Solve For x : $-2x - 5 = -2x + 8$

$$\begin{array}{r} -2x - 5 = -2x + 8 \text{ (add } 2x \text{ to} \\ +2x \quad +2x \quad \text{both sides)} \end{array}$$

$$-5 = 8$$

the x terms have dropped out
and the the remaining statement
is FALSE so there are no numbers
that work and the answer is
No Solution

Equations With All Numbers As A Solution

When you are getting the x terms on the same side of an equation and the **x terms cancel out** leaving you a statement with only numbers in it and **that statement is TRUE** then the answer is

All Real Numbers (ARN)

Example 3

Solve for x : $5x - 8 = 5x - 8$

$$\begin{array}{r} 5x - 8 = 5x - 8 \text{ (subtract } 5x \text{ from} \\ -5x \quad -5x \quad \text{both sides)} \end{array}$$

$$-8 = -8$$

the x terms have dropped out
and the the remaining statement
is TRUE so the answer is
All Real Numbers Work

Example 4

Solve for x : $-3x + 4 = -3x + 4$

$$\begin{array}{r} -3x + 4 = -3x + 4 \text{ (add } 3x \text{ to} \\ +3x \quad +3x \quad \text{both sides)} \end{array}$$

$$4 = 4$$

the x terms have dropped out
and the the remaining statement
is TRUE so the answer is
All Real Numbers Work

Distributive Equations

In some equations there may a distributive step included. If that is the case then **perform the distributive step first** and then proceed to solve for x .

Example 1

$$-2(x - 3) = -8$$

distribute the -2

$$-2x + 6 = -8$$

subtract 6 from both sides

$$-2x + 6 = -8$$

$$-6 \quad -6$$

$$-2x = -14$$

divide both sides by -2

$$\frac{-2x}{-2} = \frac{-14}{-2}$$

$$x = 7$$

check: $x = 7$

$$-2(7 - 3) = -8$$

$$-2(4) = -8$$

$$-8 = -8$$

Example 2

$$2(3x - 4) = 10 + 4x$$

distribute the 2

$$6x - 8 = 10 + 4x$$

subtract $-4x$ from both sides

$$6x - 8 = 10 + 4x$$

$$-4x \quad -4x$$

$$2x - 8 = 10$$

add 8 to both sides

$$2x - 8 = 10$$

$$+8 \quad +8$$

$$2x = 18$$

divide both sides by 2

$$\frac{2x}{2} = \frac{18}{2}$$

$$x = 9$$

Example 3

$$-3(x + 2) = 10 - x$$

distribute the -3

$$-3x - 6 = 10 - x$$

add $3x$ to both sides

$$-3x - 6 = 10 - x$$

$$+3x \quad +3x$$

$$-6 = 10 + 2x$$

subtract 10 from both sides

$$-6 = 10 + 2x$$

$$-10 \quad -10$$

$$-16 = 2x$$

divide both sides by 2

$$\frac{-16}{2} = \frac{2x}{2}$$

$$-8 = x$$

Like Terms Equations

In some equations there may **like terms on the same side of the equation**. If that is the case then **combine like terms on the same side of the equation first** and then proceed to solve for x .

Example 1

$$3x + 2x - 4 = 22 + 4$$

combine the $3x$ and $2x$ on the left
combine the 22 and 4 on the right

$$5x - 4 = 26$$

add 4 to
both sides of the equation

$$\begin{array}{r} 5x - 4 = 26 \\ + 4 \quad + 4 \end{array}$$

$$5x = 30$$

divide both sides of the
equation by 5

$$\frac{5x}{5} = \frac{30}{5}$$

$$x = 6$$

check : $x = 6$

$$3x + 2x - 4 = 22 + 4$$

$$3(6) + 2(6) - 4 = 22 + 4$$

$$18 + 12 - 4 = 26$$

$$26 = 26$$

Example 2

$$-15 + 6 + 2x = -3 + 3x + x$$

combine the -15 and 6 on the left
combine the $3x$ and x on the right

$$-9 + 2x = -3 + 4x$$

subtract $2x$ from
both sides of the equation

$$\begin{array}{r} -9 + 2x = -3 + 4x \\ - 2x \quad - 2x \end{array}$$

$$-9 = -3 + 2x$$

add 3 to
both sides of the equation

$$\begin{array}{r} -9 = -3 + 2x \\ + 3 \quad + 3 \end{array}$$

$$-6 = 2x$$

divide both sides of
the equation by 2

$$\frac{-6}{2} = \frac{2x}{2}$$

$$-3 = x$$