Section 10 – 3:
Solving Quadratic Equations by Completing the Square

Solve the quadratic equation \(x^2 \pm bx \pm \text{constant} = 0\)

**Step 1:** Get the equation into \(x^2 \pm bx = \text{constant}\) form by moving the constant from the left side of the equation to the right side. Do this by adding or subtracting the constant to both sides of the equation.

**Step 2:** The equation will now be in the form \(x^2 \pm bx = \text{constant}\). The number in front of the \(x\) term is labeled \(b\). Compute \(\left(\frac{1}{2} \cdot b\right)^2\) and add this value to both sides of the equation.

You will now have an equation in the form \(x^2 \pm bx + c = \text{constant}\)

**Step 3:** The trinomial \(x^2 \pm bx + c\) can be written as a perfect square \(\left(x - \frac{1}{2}b\right)^2\)

You will now have an equation in the form \((x \pm \text{number})^2 = \text{constant}\) where the number is \(\left(\frac{1}{2} \cdot b\right)\)

**Step 4:** Take the square root of each side. The Square Root Property requires the use of a \(\pm\) with the square root of the constant. You will now have an equation of the form \((x \pm c) = \pm \sqrt{\text{constant}}\)

**Step 5:** You will now solve for \(x\) by moving the number \(c\) from the left side of the equation to the right side by addition or subtraction. \(x = \pm c \pm \sqrt{\text{constant}}\)
Example 1
Solve the quadratic equation

\[ x^2 - 8x + 6 = 0 \]

get the equation into \( x^2 \pm bx = \pm c \) by subtracting 6 from both sides

\[ x^2 - 8x = -6 \]

\[ b = -8 \]

add \( \left( \frac{1}{2} \cdot b \right)^2 \) to both sides of the equation

\[ \left( \frac{1}{2} \cdot -8 \right)^2 = (-4)^2 = 16 \]

add 16 to both sides of the equation

\[ x^2 - 8x + 16 = -6 + 16 \]

\[ x^2 - 8x + 16 = 10 \]

write the trinomial as a perfect square

\[ (x - 4)^2 = 10 \]

take the square root of both sides

use \( \pm \sqrt{10} \)

\[ x - 4 = \pm \sqrt{10} \]

add 4 to both sides

\[ x = 4 \pm \sqrt{10} \]

Example 2
Solve the quadratic equation

\[ x^2 + 6x - 5 = 0 \]

get the equation into \( x^2 \pm bx = \pm c \) by adding 5 to both sides

\[ x^2 + 6x = 5 \]

\[ b = 6 \]

add \( \left( \frac{1}{2} \cdot b \right)^2 \) to both sides of the equation

\[ \left( \frac{1}{2} \cdot 6 \right)^2 = (3)^2 = 9 \]

add 9 to both sides of the equation

\[ x^2 + 6x + 9 = 5 + 9 \]

\[ x^2 + 6x + 9 = 14 \]

write the trinomial as a perfect square

\[ (x + 3)^2 = 14 \]

take the square root of both sides

use \( \pm \sqrt{14} \)

\[ x + 3 = \pm \sqrt{14} \]

subtract 3 from both sides

\[ x = -3 \pm \sqrt{14} \]
**Example 3**

Solve the quadratic equation

\[ x^2 - 10x + 7 = 0 \]

get the equation into

\[ x^2 \pm bx = \pm c \] by subtracting 7 from both sides

\[ x^2 - 10x = -7 \]

\[ b = -10 \]

\[ \text{add} \left( \frac{1}{2} \cdot b \right)^2 \]

to both sides of the equation

\[ \left( \frac{1}{2} \cdot -10 \right)^2 = (-5)^2 = 25 \]

\[ \text{add} 25 \]

to both sides of the equation

\[ x^2 - 10x + 25 = -7 + 25 \]

\[ x^2 - 10x + 25 = 18 \]

write the trinomial as a perfect square

\[ (x - 5)^2 = 18 \]

take the square root of both sides

use \( \pm \sqrt{18} = \pm \sqrt{2 \cdot 9} = \pm 3\sqrt{2} \)

\[ x - 5 = \pm 3\sqrt{2} \]

\[ \text{add} 5 \] to both sides

\[ x = 5 \pm 3\sqrt{2} \]

**Example 4**

Solve the quadratic equation

\[ x^2 + 4x - 8 = 0 \]

get the equation into

\[ x^2 \pm bx = \pm c \] by adding 8 to both sides

\[ x^2 + 4x = 8 \]

\[ b = 4 \]

\[ \text{add} \left( \frac{1}{2} \cdot b \right)^2 \]

to both sides of the equation

\[ \left( \frac{1}{2} \cdot 4 \right)^2 = (2)^2 = 4 \]

\[ \text{add} 4 \]

to both sides of the equation

\[ x^2 + 4x + 8 = 4 + 8 \]

\[ x^2 + 4x + 8 = 12 \]

write the trinomial as a perfect square

\[ (x + 2)^2 = 12 \]

take the square root of both sides

use \( \pm \sqrt{12} = \pm \sqrt{4 \cdot 3} = \pm 2\sqrt{3} \)

\[ x + 2 = \pm 2\sqrt{3} \]

\[ \text{subtract} 2 \] from both sides

\[ x = -2 \pm 2\sqrt{3} \]
Example 5

\[ x^2 + 3x - 2 = 0 \]

get the equation into

\[ x^2 \pm bx = \pm c \]

by adding 2 to both sides

\[ x^2 + 3x = 2 \]

\[ b = 3 \]

add \( \left( \frac{1}{2} \cdot b \right)^2 \) to both sides of the equation

\[ \left( \frac{1}{2} \cdot 3 \right)^2 = \left( \frac{3}{2} \right)^2 = \frac{9}{4} \]

add \( \frac{9}{4} \) to both sides of the equation

\[ x^2 + 3x + \frac{9}{4} = 2 + \frac{9}{4} \]

\[ x^2 + 4x + \frac{9}{4} = \frac{17}{4} \]

write the trinomial as a perfect square

\[ \left( x + \frac{3}{2} \right)^2 = \frac{17}{4} \]

take the square root of both sides

\[ \left( x + \frac{3}{2} \right) = \sqrt{\frac{17}{4}} \]

\[ x + \frac{3}{2} = \pm \frac{\sqrt{17}}{2} \]

subtract \( \frac{3}{2} \) from both sides

\[ x = -\frac{3}{2} \pm \frac{\sqrt{17}}{2} = -\frac{3 \pm \sqrt{17}}{2} \]