Section 10 – 3:

Solving Quadratic Equations by Completing the Square

Solve the quadratic equation $x^2 \pm bx \pm \text{constant} = 0$

Step 1: Get the equation into $x^2 \pm bx = \text{constant}$ form by moving the constant from the left side of the equation to the right side. Do this by adding or subtracting the constant to both sides of the equation.

Step 2 : The equation will now be in the form $x^2 \pm bx = \text{constant}$ The number in front of the x term is labeled b. Compute $\left(\frac{1}{2} \bullet b\right)^2$ and add this value to both sides of the equation. You will now have an equation in the form $x^2 \pm bx + c = \text{constant}$

Step 3 : The trinomial $x^2 \pm bx + c$ can be written as a perfect square $\left(x - \frac{1}{2}b\right)^2$ You will now have an equation in the form $(x \pm \text{number})^2 = \text{constant}$ where the number is $\left(\frac{1}{2} \bullet b\right)$

Step 4 : Take the square root of each side. The Square Root Property requires the use of a $\pm$ with the square root of the constant You will now have an equation of the form $(x \pm c) = \pm\sqrt{\text{constant}}$

Step 5 : You will now solve for x by moving the number c from the left side of the equation to the right side by addition or subtraction. $x = \pm c \pm \sqrt{\text{constant}}$

Example 1

Solve the quadratic equation

$$x^2 - 8x + 6 = 0$$

get the equatioon into

$x^2 \pm bx = \pm c$ by

subtracting 6 from both sides

$$x^2 - 8x = -6$$

$$b = -8$$

add $\left(\frac{1}{2} \bullet b\right)^2$

to both sides of the equation

$$\left(\frac{1}{2} \bullet -8\right)^2 = (-4)^2 = 16$$

add 16

to both sides of the equation

$$x^2 - 8x + 16 = -6 + 16$$

$$x^2 - 8x + 16 = 10$$

write the trinominal as a perfect square

$$(x - 4)^2 = 10$$

take the square root of both sides

use $\pm\sqrt{10}$

$$x - 4 = \pm\sqrt{10}$$

add 4 to both sides

$$x = 4 \pm \sqrt{10}$$

Example 2

Solve the quadratic equation

$$x^2 + 6x - 5 = 0$$

get the equatioon into

$x^2 \pm bx = \pm c$ by

adding 5 to both sides

$$x^2 + 6x = 5$$

$$b = 6$$

add $\left(\frac{1}{2} \bullet b\right)^2$

to both sides of the equation

$$\left(\frac{1}{2} \bullet 6\right)^2 = (3)^2 = 9$$

add 9

to both sides of the equation

$$x^2 + 6x + 9 = 5 + 9$$

$$x^2 + 6x + 9 = 14$$

write the trinominal as a perfect square

$$(x + 3)^2 = 14$$

take the square root of both sides

use $\pm\sqrt{14}$

$$x + 3 = \pm\sqrt{14}$$

subtract 3 from both sides

$$x = -3 \pm \sqrt{14}$$

Example 3

Solve the quadratic equation

$$x^2 - 10x + 7 = 0$$

get the equatioon into

$x^2 \pm bx = \pm c$ by

subtracting 7 from both sides

$$x^2 - 10x = -7$$

$$b = -10$$

add $\left(\frac{1}{2} \bullet b\right)^2$

to both sides of the equation

$$\left(\frac{1}{2} \bullet -10\right)^2 = (-5)^2 = 25$$

add 25

to both sides of the equation

$$x^2 - 10x + 25 = -7 + 25$$

$$x^2 - 10x + 25 = 18$$

write the trinominal as a perfect square

$$(x - 5)^2 = 18$$

take the square root of both sides

use $\pm\sqrt{18} = \pm\sqrt{2 \bullet 9} = \pm 3\sqrt{2}$

$$x - 5 = \pm 3\sqrt{2}$$

add 5 to both sides

$$x = 5 \pm 3\sqrt{2}$$

Example 4

Solve the quadratic equation

$$x^2 + 4x - 8 = 0$$

get the equatioon into

$x^2 \pm bx = \pm c$ by

adding 8 to both sides

$$x^2 + 4x = 8$$

$$b = 4$$

add $\left(\frac{1}{2} \bullet b\right)^2$

to both sides of the equation

$$\left(\frac{1}{2} \bullet 4\right)^2 = (2)^2 = 4$$

add 4

to both sides of the equation

$$x^2 + 4x + 8 = 4 + 8$$

$$x^2 + 4x + 8 = 12$$

write the trinominal as a perfect square

$$(x + 2)^2 = 12$$

take the square root of both sides

use $\pm\sqrt{12} = \pm\sqrt{4 \bullet 3} = \pm 2\sqrt{3}$

$$x + 2 = \pm 2\sqrt{3}$$

subtract 2 from both sides

$$x = -2 \pm 2\sqrt{3}$$

Example 5

$$x^2 + 3x - 2 = 0$$

get the equatioon into

$x^2 \pm bx = \pm c$ by

adding 2 to both sides

$$x^2 + 3x = 2$$

$$b = 3$$

add $\left(\frac{1}{2} \bullet b\right)^2$ to both sides of the equation

$$\left(\frac{1}{2} \bullet 3\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

add 9 / 4to both sides of the equation

$$x^2 + 3x + \frac{9}{4} = 2 + \frac{9}{4}$$

$$x^2 + 4x + \frac{9}{4} = \frac{17}{4}$$

write the trinominal as a perfect square

$$\left(x + \frac{3}{2}\right)^2 = \frac{17}{4}$$

take the square root of both sides

$$\left(x + \frac{3}{2}\right) = \sqrt{\frac{17}{4}}$$

$$x + \frac{3}{2} = \pm \frac{\sqrt{17}}{2}$$

subtract 3 / 2 from both sides

$$x = -\frac{3}{2} \pm \frac{\sqrt{17}}{2} = \frac{-3 \pm \sqrt{17}}{2}$$

