

Section 1 – 5: Evaluating Algebraic Expressions

Algebraic Expressions

An algebraic expression is any expression that combines numbers and letters with a set of operations on those numbers and letters. **An expression does not contain an = sign.**

Vocabulary for Algebraic Expressions

Term: Each part of an expression that is separated by a + or – sign is one **term** in the expression.

In the expression $2x + 5$ The $2x$ and the $+ 5$ are each separate terms

In the expression $3x^2 - 2x - 7$ the $3x^2$ and the $-2x$ and the -7 are each separate terms

Constant Term: A number by itself with no variable is a **constant term**. The term is called **constant** because its value is known and stays the same.

In $2x + 5$ the 5 is a constant term

In $3x^2 - 2x - 7$ the -7 is a constant term

Variable Term: Each letter is called a **variable**. Each term with a variable is a variable term

In $2x + 5$ the $2x$ is a variable term

In $3x^2 - 2x - 7$ the $3x^2$ and the $-2x$ are variable terms

Coefficient of a Term: The number in front of a variable is called a **coefficient**.

In $2x + 5$ the 2 is a coefficient of x

In $3x^2 - 7$ the 3 is a coefficient of x^2

Exponent: The number above the variable is called an **exponent** (or power) of the variable.

$3x^2$ the variable x has an exponent of 2

$2x^3$ the variable x has an exponent of 3

$5x$ the variable x has with an exponent of 1. If the variable has an exponent of 1 it is not shown.

Example: $-4x^2 + 2y - 8$

The expression $-4x^2 + 2y - 8$ has 3 terms. $-4x^2$ and $+2y$ and -8 are each separate terms.

The $-4x^2$ is a variable term with a coefficient of -4 and the variable x has an exponent of 2

The $+2y$ is a variable term with a coefficient of $+2$ and the variable y has an exponent of 1

The -8 is a constant term.

Reading an Algebraic Expression

This chapter will involve expressions or terms that have a constant in front of a variable. The variable may also have a number above it as an exponent. Each expression is read by reading the constant first and then the variable and then the power. A constant and a variable written next to each other **mean to multiply** the constant times the variable. Variables written next to each other **mean to multiply** one variable times the other variable.

Example 1

$5x$ is read
five x
and means
5 times x

Example 2

$-2x$ is read
negative two x
and means
 -2 times x

Example 3

$-x$ is read
negative x
and means
 -1 times x

Example 4

$5x^2y$ is read
5 x squared y
and means
5 times x times x times y

Example 5

$-xy^2$ is read
negative $x y$ squared
and means
 -1 times x times y times y

Example 6

$4x^3$ is read
four x to the third
and means
4 times x times x times x

Evaluation of Algebraic Expressions

If the value for each variable is known then the value for the entire expression can be found. A given value for each variable can be **substituted** in for that letter to create expressions like the ones in Section 4. The value of the expression is then determined by using the rules for the order of operations PEMDAS.

The process of **substituting** values into the expression for each variable and then determining the number value for the expression is called **Evaluating the Expression**.

The process of **substituting** values into the expression for each variable involves **creating a set of parenthesis for each variable letter** and then **substituting** the values for each letter into the parenthesis. The final process of **Evaluating the Expression** involves simplifying the resulting expression using the rules for the order of operations PEMDAS. The **Steps to Evaluate Algebraic Expressions** are given next.

Steps to Evaluate an Algebraic Expression

Step 1. Rewrite the expression with each variable (letter) inside a separate parenthesis.

Step 2. Replace each variable with the value given for it (substitution).

Step 3. Use the order of operation rules (PEMDAS) to find the value of the expression.

If $A = -3$ and $B = 2$ and $C = 4$ then evaluate the expression.

Example 1

$$\begin{aligned} 4AB \text{ means } 4(A)(B) \\ = 4(-3)(2) = -24 \end{aligned}$$

Example 2

$$\begin{aligned} -BC \text{ means } -1(B)(C) \\ = -1(2)(4) = -8 \end{aligned}$$

Example 3

$$\begin{aligned} 4A^2 \text{ means } 4(A)^2 \\ = 4(-3)^2 = 4(9) = 36 \end{aligned}$$

If $A = -2$ and $B = -3$ and $C = 2$ then evaluate the expression.

Example 4

$$\begin{aligned} 2A + 3B \\ \text{means } 2(A) + 3(B) \\ 2(-2) + 3(-3) \\ -4 - 9 \\ = -13 \end{aligned}$$

Example 5

$$\begin{aligned} 4A - C \\ \text{means } 4(A) - (C) \\ 4(-2) - (2) \\ -8 - 2 \\ = -10 \end{aligned}$$

Example 6

$$\begin{aligned} 5B - 2A^2 \\ \text{means } 5(B) - 2(A)^2 \\ 5(-3) - 2(-2)^2 \\ -15 - 8 \\ = -23 \end{aligned}$$

If $A = -3$ and $B = 2$ and $C = 4$ then evaluate the expression.

Example 7

$$\begin{aligned} \frac{-5C}{A+B} \text{ means } \frac{-5(4)}{-3+(2)} \\ = \frac{-20}{-3+2} \\ = \frac{-20}{-1} = -20 \end{aligned}$$

Example 8

$$\begin{aligned} \frac{-3C+A}{A-B} \text{ means } \frac{-3(4)+(-3)}{-3-(2)} \\ = \frac{-12-3}{-3-2} \\ = \frac{-15}{-5} = 3 \end{aligned}$$

Example 9

$$\begin{aligned} \frac{5C-3B}{B-C} \text{ means } \frac{5(4)-3(2)}{2-(4)} \\ = \frac{20-6}{2-4} \\ = \frac{14}{-2} = -7 \end{aligned}$$

Evaluation of Expressions with x and y

Many expressions in algebra use x and y as the variables instead of the A,B or C used so far in this chapter. It is also common to change the values for the variables with each problem rather than keeping the values the same for several problems. The problems below are completed the same way as the earlier problems but use x and y variables instead and change the value of x and y in each problem.

Example 1

Evaluate $-3x + 2$
for $x = -2$

$$-3(-2) + 2$$

$$= 6 + 2$$

$$= 8$$

Example 2

Evaluate $-2x^2 - x$
for $x = -3$

$$-2(-3)^2 - (-3)$$

$$= -2(9) + 3$$

$$= -18 + 3$$

$$= -15$$

Example 3

Evaluate $2x^2 - 5x$
for $x = 4$

$$2(4)^2 - 5(4)$$

$$= 2(16) - 5(4)$$

$$= 32 - 20$$

$$= 12$$

Example 4

Evaluate $-x^2 - 3x - 5$ for $x = -4$

$$-(-4)^2 - 3(-4) - 5$$

$$= -(16) - 3(-4) - 5$$

$$= -16 + 12 - 5$$

$$= -9$$

Example 5

Evaluate $-2xy + 2y^2$ for $x = -2$ and $y = 3$

$$-2(-2)(3) + 2(3)^2$$

$$= -2(-2)(3) + 2(9)$$

$$= 12 + 18$$

$$= 30$$

Example 6

Evaluate $-8x + 6y$ for $x = \frac{3}{4}$ and $y = \frac{5}{2}$

$$\frac{-8}{1}\left(\frac{3}{4}\right) + \frac{6}{1}\left(\frac{5}{2}\right)$$

$$= \frac{-24}{4} + \frac{30}{2}$$

$$= -6 + 15$$

$$= 9$$

Example 7

Evaluate $\frac{x-y}{2x+y}$ for $x = 3$ and $y = -4$

$$\frac{3 - (-4)}{2(3) + (-4)}$$

$$= \frac{3 + 4}{6 - 4}$$

$$= \frac{7}{2}$$

Evaluation of Formulas Introduction

There are many problems in mathematics that use formulas with variables to express the relationship between the different variables in the formula. The following are some of the most common ones found in Algebra and Geometry courses.

Convert Temperature in degree Fahrenheit to degrees Celsius

The temperature in **Celsius** is given by $C = \frac{5(F - 32)}{9}$ where C is the temperature in degrees Celsius and F is the temperature in Degrees Fahrenheit.

Example 1

Find the temperature in C if

$$C = \frac{5(F - 32)}{9}$$

and $F = 77$ degrees F

$$C = \frac{5(77 - 32)}{9}$$

$$C = \frac{5(45)}{9} = 25^{\circ}\text{C}$$

Example 2

Find the temperature in C if

$$C = \frac{5(F - 32)}{9}$$

and $F = 59$ degrees F

$$C = \frac{5(59 - 32)}{9}$$

$$C = \frac{5(27)}{9} = 15^{\circ}\text{C}$$

Convert Temperature in degrees Celsius to degrees Fahrenheit

The Temperature in **Fahrenheit** is given by $F = \frac{9C}{5} + 32$ where F is the temperature in degrees Fahrenheit and C is the temperature in Degrees Celsius.

Example 3

Find the temperature in F if

$$F = \frac{9C}{5} + 32$$

and $C = 75$ degrees C

$$F = \frac{9 \cdot 75}{5} + 32$$

$$F = 135 + 32 = 167^{\circ}\text{F}$$

Example 4

Find the temperature in F if

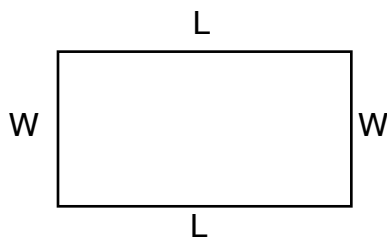
$$F = \frac{9C}{5} + 32$$

and $C = 35$ degrees C

$$F = \frac{9 \cdot 35}{5} + 32$$

$$F = 63 + 32 = 95^{\circ}\text{F}$$

The Perimeter of a Rectangle



The **perimeter of a rectangle** is expressed by the formula $P = 2W + 2L$ where P is the perimeter, W is the width of the rectangle and L is the length of the rectangle. The answer will be in length units: inches, feet, yards.

Example 5

Find the perimeter of a rectangle if

$$P = 2W + 2L$$

and $W = 3$ in., $L = 4$ in.

$$P = 2(3) + 2(4)$$

$$P = 6 + 8$$

$$P = 14 \text{ inches}$$

Example 6

Find the perimeter of a rectangle if

$$P = 2W + 2L$$

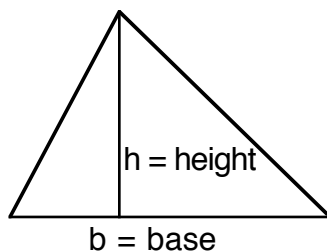
and $W = 6$ yds., $L = 5$ yds.

$$P = 2(6) + 2(5)$$

$$P = 12 + 10$$

$$P = 22 \text{ yards}$$

The Area of a Triangle



The **Area of a triangle** is expressed by the formula $A = \frac{1}{2}bh$ where A is the Area, b is the base of the triangle and h is the height of the triangle. The answer will be in square units: square inches, square feet, square yards.

Example 7

Find the area of a triangle if

$$A = \frac{1}{2}bh$$

and $b = 3$ in., $h = 6$ in.

$$A = \frac{1}{2}(3)(6)$$

$$A = 9 \text{ sq. in.}$$

Example 8

Find the area of a triangle if

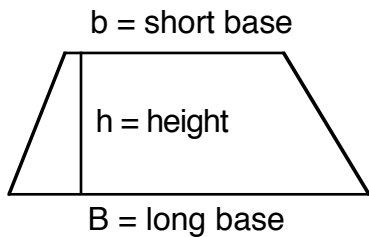
$$A = \frac{1}{2}bh$$

and $b = 12$ ft., $h = 5$ ft.

$$A = \frac{1}{2}(12)(5)$$

$$A = 30 \text{ sq. ft.}$$

The Area of a Trapezoid



The **Area of a trapezoid** is expressed by the formula

$A = \frac{h(B+b)}{2}$ where A is the Area, h is the **height**, B is the **long Base** of the trapezoid and b is the **short base** of the trapezoid.

The answer will be in square units: square inches, square feet, square yards.

Example 9

Find the area of a trapezoid if

$$A = \frac{h(B+b)}{2}$$

and $h = 3$ in., $B = 4$ in. $b = 6$ in.

$$A = \frac{3(4+6)}{2}$$

$$A = \frac{3(10)}{2}$$

$$A = \frac{30}{2}$$

$$A = 15 \text{ sq. in.}$$

Example 10

Find the area of a trapezoid if

$$A = \frac{h(B+b)}{2}$$

and $h = 6$ ft., $B = 2$ ft. $b = 7$ ft.

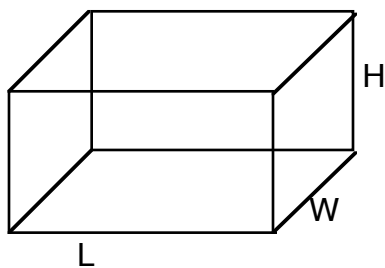
$$A = \frac{6(2+7)}{2}$$

$$A = \frac{6(9)}{2}$$

$$A = \frac{54}{2}$$

$$A = 27 \text{ sq. ft.}$$

The Volume of a Rectangular Solid



The **Volume of a Rectangular Solid** is given by $V = L \cdot W \cdot H$ where V is the volume of the solid, L is the length of the solid, W is the width of the solid and H is the Height of the solid. The answer will be in cubic units: cubic inches, cubic feet, cubic yards.

Example 11

Find the volume of a Rectangular Solid

if $V = L \cdot W \cdot H$

and $L = 4$ in., $W = 3$ in., $H = 10$ in.

$$V = 4 \cdot 3 \cdot 10$$

$$= 120 \text{ cubic inches}$$

Example 12

Find the volume of a Rectangular Solid

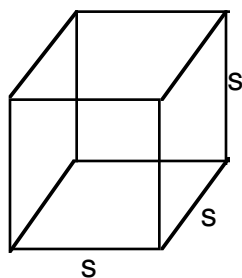
if $V = L \cdot W \cdot H$

and $L = 5$ yds., $W = 2$ yds., $H = 6$ yds.

$$V = 5 \cdot 2 \cdot 6$$

$$= 60 \text{ cubic yards}$$

The Volume of a Cube



A cube is a rectangular solid with every face being a square.

The **Volume of a Cube** is given by $V = s^3$ where V is the volume of the cube and s is the length of every side of the cube. The answer will be in cubic units: cubic inches, cubic feet, cubic yards.

Example 13

Find the volume of a Cube if

$$V = s^3$$

$$\text{and } s = 4 \text{ feet}$$

$$V = 4^3$$

$$V = 64 \text{ cubic feet}$$

Example 14

Find the volume of a Cube if

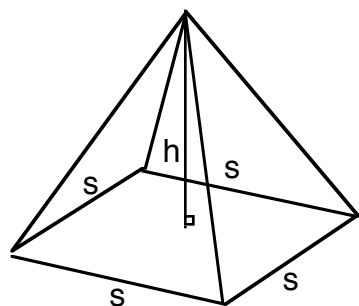
$$V = s^3$$

$$\text{and } s = \frac{1}{2} \text{ inches}$$

$$V = \left(\frac{1}{2}\right)^3$$

$$V = \frac{1}{8} \text{ cubic inches}$$

The Volume of a Square Pyramid



The **Volume of a Square Pyramid** is given by $V = \frac{1}{3} \cdot s^2 \cdot h$

where V is the volume of the Square Pyramid, s is the side of the square base and h is the height of the Square Pyramid. The answer will be in cubic units: cubic inches, cubic feet, cubic yards.

Example 15

Find the volume of a Square Pyramid

$$\text{if } V = \frac{1}{3} \cdot s^2 \cdot h$$

$$\text{and } s = 3 \text{ inches and } h = 2 \text{ inches}$$

$$V = \frac{1}{3} \cdot 3^2 \cdot 2$$

$$V = \frac{1}{3} \cdot 9 \cdot 2$$

$$V = 6 \text{ cubic inches}$$

Example 16

Find the volume of a Square Pyramid

$$\text{if } V = \frac{1}{3} \cdot s^2 \cdot h$$

$$\text{and } s = 1 \text{ inch and } h = 9 \text{ inches}$$

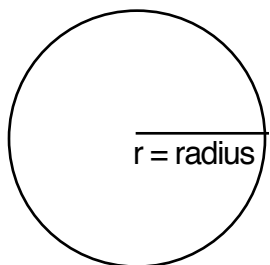
$$V = \frac{1}{3} \cdot 1^2 \cdot 9$$

$$V = \frac{1}{3} \cdot 1 \cdot 9$$

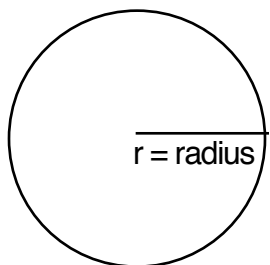
$$V = 3 \text{ cubic inches}$$

Evaluation of Geometric Formulas with π Introduction

There are many shapes in geometry that involve a circle. The formulas for the Circumference, Area, and Volume of shapes that involve circles contain the symbol π . The symbol π stands for an irrational number called pi. An irrational number is a decimal number that **never ends or repeats**. If you want an answer that is **exact** the answer **must must be written with the symbol π in it**. It is common to **estimate** an answer by using a decimal number that is **close to π** . The decimal number 3.14 is often used in place of π . We use the \approx instead of $=$ to show that the answer is an **approximation** and not not exact.



The **circumference of a circle** is the distance around the circle and is found by the formula $C = 2 \cdot \pi \cdot r$ where C is the circumference of the circle, r is the radius of the circle and π is an irrational number. The answer will be in length units: inches, feet, or yards.



The **area of a circle** is given by $A = \pi \cdot r^2$ where A is the area of the circle, r is the radius of the circle and π is an irrational number. The answer will be in square units: square inches, square feet, or square yards.

Example 1

Find the exact circumference of a circle

$$\text{if } C = 2 \cdot \pi \cdot r$$

and $r = 8$ yards

$$C = 2 \cdot \pi \cdot 8$$

$$P = 16\pi \text{ yards}$$

Example 2

Find the exact area of a circle

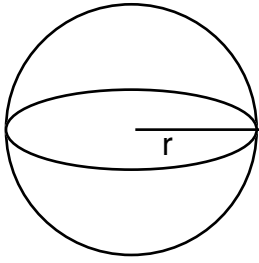
$$\text{if } A = \pi \cdot r^2$$

and $r = 6$ ft.

$$A = \pi \cdot 6^2$$

$$A = 36\pi \text{ sq. ft.}$$

The Volume of a Sphere



The **volume of a sphere** is given by $V = \frac{4}{3} \cdot \pi \cdot r^3$

where V is the volume of the sphere, r is the radius of the sphere and π is an irrational number. The answer will be in cubic units: cubic inches, cubic feet, or cubic yards.

Example 3

Find the exact volume of a sphere

$$\text{if } V = \frac{4}{3} \cdot \pi \cdot r^3$$

and $r = 3$ inches

$$V = \frac{4}{3} \cdot \pi \cdot 3^3$$

$$V = \frac{4}{3} \cdot \pi \cdot 27$$

$$V = 36\pi \text{ cubic inches}$$

Example 4

Find the exact volume of a sphere

$$\text{if } V = \frac{4}{3} \cdot \pi \cdot r^3$$

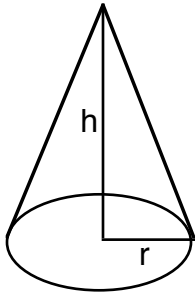
and $r = 2$ feet

$$V = \frac{4}{3} \cdot \pi \cdot 2^3$$

$$V = \frac{4}{3} \cdot \pi \cdot 8$$

$$V = \frac{32\pi}{3} \text{ cubic feet}$$

The Volume of a Cone



The **volume of a cone** is given by $V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$ where V is the volume of the cone, r is the radius of the base, h is the height of the cone and π is an irrational number. The answer will be in cubic units: cubic inches, cubic feet, or cubic yards.

Example 5

Find the exact volume of a cone

$$\text{if } V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$$

and $r = 4$ ft. and $h = 3$ ft.

$$V = \frac{1}{3} \cdot \pi \cdot 4^2 \cdot 3$$

$$V = \frac{1}{3} \cdot \pi \cdot 48$$

$$V = 16\pi \text{ cubic feet}$$

Example 6

Find the exact volume of a cone

$$\text{if } V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$$

and $r = 6$ inch and $h = 1$ inch

$$V = \frac{1}{3} \cdot \pi \cdot 6^2 \cdot 1$$

$$V = \frac{1}{3} \cdot \pi \cdot 36$$

$$V = 12\pi \text{ cubic inches}$$