Section 1 – 4: The Order of Operations

There is an agreed upon order in which the operations of exponents, multiplication, division, addition and subtraction are performed if there are several different operations in one expression.

We use the mnemonic P.E.M.D.A.S to help us remember the order of operations.

Please **E**xcuse **M**y **D**ear **A**unt **S**ally

1. **P** Perform the operations inside the parentheses and get **one number** inside the parenthesis.
2. **E** Exponents are next. Multiply the base times itself the number of times as the exponent.
3. **M,D** Perform all the simple multiply and/or divide problems.
4. **A,S** Perform the addition and/or subtraction of signed numbers.

Parentheses

The first thing we do is to perform any operations that are **inside the parentheses**. We **keep the answer** to this step **inside the parentheses**. If there is just one number inside the parentheses we do not perform this step.

Example 1

$(5-7)$

is replaced by

(-2)

Example 2

$5(-3-6)$

is replaced by

$5(-9)$

Example 3

$(9-2)^2$

is replaced by

$(7)^2$

Example 4

$6(5-7)-(3-7)$

is replaced by

$6(-2)-(-4)$

Example 5

$2(10-7)^2+3(6-1)^2$

is replaced by

$2(3)^2+3(5)^2$

Example 6

$$\frac{2(10-7)^2}{3(6-1)^2}$$

is replaced by

$$\frac{2(3)^2}{3(5)^2}$$

Exponents

The next operation involves any number that has an **exponent** above it. You multiply the number on the bottom, **the base**, times itself the number of times as the exponent.

Example 1

$5^2 = (5)(5) = 25$

Example 2

$3^2 = (3)(3) = 9$

Example 3

$(4)^2 = (4)(4) = 16$

Example 4

$(7)^2 = (7)(7) = 49$

Example 5

$2^3 = (2)(2)(2) = 8$

Example 6

$2^4 = (2)(2)(2)(2) = 16$

Important Note: The negative sign in front of the base **is included** with the base if the negative sign is inside the parentheses

Example 7

$(-3)^2 = (-3)(-3) = 9$

Example 8

$(-7)^2 = (-7)(-7) = 49$

Example 9

$(-2)^3 = (-2)(-2)(-2) = -8$

Important Note: A negative sign **in front** of the base is **not included with the base** if the base is not inside a parentheses. The negative sign in front of the base is understood to mean multiply the products of the base by –1. **This is a very common source for errors in this chapter.**

Example 10

$-2^2 = -1(2)^2 = -4$

the – sign is NOT included with the 2

Example 11

$-3^2 = -1(3)^2 = -9$

the – sign is NOT included with the 3

Example 12

$-4^2 = -1(4)^2 = -16$

the – sign is NOT included with the 4

Multiply and Divide

The next thing we do is to find any number that is in front of a parenthesis and perform the multiplication. We also find all the division problems and perform the division. For this section of the chapter the division sign will be written using the $\div$ symbol. 8 divided by 4 will be written as $8 \div 4$. Future sections will replace the $\div$ symbol with a fraction.

Example 1

$-8 + 5(2)$

the product $5(2)$ is found first

$= -8 + 10$

Example 2

$-3(4) - 7$

the product $-3(4)$ is found first

$= -12 - 7$

Example 3

$-8 - 3(7)$

the product $-3(7)$ is found first

$= -8 - 21$

Example 4

$-4 - 15 \div 3$

the division $-15 \div 3$ is found first

$= -4 - 5$

Example 5

$-8 \div 2 - 9$

the division $-8 \div 2$ is found first

$= -4 - 9$

Example 6

$7 + 20 \div 10$

the division $20 \div 10$ is found first

$= 7 + 2$

Example 7

$-4(6) + 5(2)$

both products $-4(6)$ and $5(2)$ are found first

$= -24 + 10$

Example 8

$4(-3) - (7)$

both products $4(-3)$ and $-(7)$ are found first

$= -12 - 7$

Example 9

$10 \div 2 + 12 \div 6$

both divisions $10 \div 2$ and $+12 \div 6$ are found first

$= 5 + 2$

Add and Subtract

After all the multiplication and division is complete there will only be numbers with a + or – sign between them. Perform the addition or subtraction of the signed numbers to complete the process.

Example 10

$$-6-4(5)$$

the product $-4(5)$ is found first

$$=-6-20$$

add / subtract is done last

$$=-26$$

Example 11

$$-3(-6)+5$$

the product $-3(-6)$ is found first

$$=18+5$$

add / subtract is done last

$$=23$$

Example 12

$$8(-3)-6(-2)$$

both products $8(-3)$ and $-6(-2)$ are found first

$$=-24+12$$

and the add / subtract is done last

$$=-12$$

Example 13

$$-10\div 2-3(-5)$$

the division $-10\div 2$ and the product $-3(-5)$ are found first

$$=-5+15$$

and the add / subtract is done last

$$=10$$

Order of Operations Examples

Example 1

$7-2(5-2)$ **P**

$7-2(3)$ **M**

$7-6$ **S**

1

Example 2

$4-5(2-6)$ **P**

$4-5(-4)$ **M**

$4+20$ **A**

24

Example 3

$4(2)+5(2-3)$ **P**

$4(2)+5(-1)$ **M**

$8-5$ **S**

3

Example 4

$(-3)^2-4^2$ **E**

$9-16$ **S**

– 7

Example 5

-4^2-3^2 **E**

$-16-9$ **A**

–25

Example 6

$-2(3)^2-2^3$ **E**

$-2(9)-8$ **M**

$-18-8$ **A**

– 26

Example 7

$(3+1)^2-2(5-2)^2$ **P**

$(4)^2-2(3)^2$ **E**

$(16)-2(9)$ **M**

$16-18$ **S**

– 2

Example 8

$2(2-5)^2+(7-3)^2$ **P**

$2(-3)^2+(4)^2$ **E**

$2(9)+(16)$ **M**

$18+16$ **A**

34

Example 9

$-(4+1)^2-3^2$ **P**

$-(5)^2-9$ **E**

$-(25)-9$ **M**

$-25-9$ **A**

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The Order of Operations with Fractions

The Numerator (top) and Denominator (bottom) of a fraction are considered **separate expressions** that are inside their own set of parentheses. The Order of Operations PEMDAS requires that we simplify the Numerator and Denominator separately and then reduce the final fraction if possible.

$$\frac{(\text{the numerator must be simplified separately})}{(\text{the denominator must be simplified separately})}$$

1. Simplify the Numerator and Denominator separately using **PEMDAS**.
2. Reduce the final fraction.

Example 10

$\frac{2(3+1)}{5(4-2)}$ **P**

$\frac{2(4)}{5(2)}$ **M**

$= \frac{8}{10}$

$= \mathbf{\frac{4}{5}}$

Example 11

$\frac{-4(2-8)}{-4+2}$ **P**

$\frac{-4(-6)}{-4+2}$ **M**

$\frac{24}{-2}$ **S**

$= -12$

Example 12

$\frac{-4(2)-2}{2(-4+2)}$ **P**

$= \frac{-4(2)-2}{2(-2)}$ **M**

$= \frac{-8-2}{-4}$ **A**

$= \frac{-10}{-4}$

$= \mathbf{\frac{5}{2}}$

